Incorporation of Flood and Other Catastrophe Model Results into Pricing and Underwriting

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Canadian Institute of Actuaries (CIA)
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AUTHORS
George Davis, FCAS, MAAA – AIR Worldwide
Stacey Gotham, FCAS, MAAA – AIR Worldwide
Alan Frith, CPCU, ARe, CEEM – AIR Worldwide
Jim Christie, FCIA, FCAS – RSM Canada
Stanley Caravaggio, FCIA, FCAS – RSM Canada

Caveat and Disclaimer

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Section 1 Introduction

This research report on the incorporation of flood and other catastrophe model results into pricing and underwriting is prepared by the authors for the Canadian Institute of Actuaries (CIA), the Society of Actuaries, and the Casualty Actuarial Society.

1.1 Overview of the Question

In the view of the CIA, there exist significant knowledge gaps for Canadian property/casualty (P&C) actuaries attempting to incorporate the results of catastrophe models into their pricing and underwriting strategies. This research report is prepared to assist the CIA's Research Executive Committee in developing recommendations for pricing methodologies and underwriting approaches that incorporate the use of catastrophe model results.

This research follows upon the recent emergence of water damage claims and climate-related perils as the largest claims costs facing property insurers in Canada. With the upwards trend in water damage claims, Canadian insurance companies have responded with the development and introduction to the market of property protection flood endorsement policies.

Catastrophe models are designed for several types of perils, i.e. earthquake, wind/hail, flood, wild fire, etc. Although this research report does discuss and present ratemaking and underwriting approaches from a general level and irrespective of peril, we also present more specific examples and research details related to applications of these methods to the flood risk category.

We set out to fulfill the needs of the CIA's research project objective through a comprehensive evaluation of different methodologies for incorporating catastrophe modeling into ratemaking and underwriting. We also set out to illustrate and produce one potential solution that is robust, predictive, and practical for the industry.

1.2 Summary of Paper

The results of our research and analysis are provided in the following sections.

Section 2 provides a brief history of the development of catastrophe models and an explanation of the general architecture and use cases of current catastrophe models.

Section 3 provides details of the current approaches for catastrophe ratemaking in Canada and contrasts those with approaches used in the United States. One key aspect of our analysis is to provide a proof-of-concept example relating to the proposed approach for flood ratemaking. Section 4 provides a summary of current flood insurance offerings in Canada. Much of the information described in these sections was determined by surveying insurers in Canada. The questions included in the survey are laid out in Appendix A — Survey Questions for Canadian Insurers.

Section 5 and Section 6 contain the major components of our research and analysis. These sections provide a detailed description of a proposed ratemaking approach, guiding the reader through each component of the process in detail. We then illustrate this approach in Section 6 with an example based on a sample company. Territorial relativities calculated during this process are included in Appendix C — Sample Company Territory Relativity Factors (Traditional).

Section 7 presents an initial exploration into additional related areas for consideration and potential variations on the proposed ratemaking approach outlined herein, such being considered outside the main scope of this research paper. Section 8 provides a summary of our research and concluding remarks on the value of the proposed ratemaking approach.

In preparing this paper the team identified and reviewed a wide range of literature touching on catastrophe rating. This material is summarized in Appendix B — Summary of Literature Review.

Section 2 Catastrophe Modeling Background

2.1 How Have Models Developed

The modern era of evaluating catastrophe perils using probabilistic catastrophe (cat) models has been developing since the 1980s. Insurers started to seriously take notice of catastrophe models in the 1990s following severe hurricanes that generated losses that were not well foreseen, especially after Hurricane Andrew hit Florida very hard in 1992.

Michael Walters and François Morin wrote about the use of catastrophe models to estimate loss costs in 1996.[1] In the same year, Burger, Fitzgerald, White, and Woods also wrote on incorporating a hurricane model into property ratemaking.[2]

Hurricane Andrew was enough of a shock to the insurance industry that companies began to pay attention to the more robust view of risk that well-constructed catastrophe models provided. Whereas the approach of catastrophe modeling was initially applied to hurricanes, it has subsequently been applied to a large (and growing) number of perils. The perils for which catastrophe modeling is well suited are those for which events are infrequent and claims for an event are correlated. This characterization is expressed in "Catastrophe Exposures and Insurance Industry Catastrophe Management Practices," published by the American Academy of Actuaries (Reference 17 in Appendix B). Today catastrophe modeling is applied to many natural perils that fit this characterization, including fairly recently inland flood, and even some unnatural perils such as terrorism and, more recently, cyber.

The expanding use of catastrophe modeling techniques has been fueled greatly by the increasing ability of computers to process vast amounts of data. Whereas early models included relatively small numbers of simulations, it is common now for models to provide tens or even hundreds of thousands of simulations, thereby providing a very robust view of scenarios that can be anticipated and allowing risk analysis at finer and finer resolution by geography or type of exposure.

As the models have evolved, in tandem with the science that is reflected in the models, acceptance of models has grown and use of models in ratemaking is standard operating procedure in many areas.

2.2 Benefits of Catastrophe Model Usage

By their nature, catastrophe models address the problem of historical data on catastrophe perils being sparse and often outdated. While an insurance organization may have diligently and effectively accumulated records of exposures and events for many decades, that historical data may have little relevance to the situation today. Patterns of behavior of a peril can change over time. Often more important is that the placement, concentration, and characteristics of exposures has changed considerably over time. One major example is that building practices in many cases have improved, which can make exposures more resilient but may also make them more expensive to reconstruct. Catastrophe models represent a current expectation of event parameters applied to a current view of exposures.

The common approach in catastrophe models is to create a set of events (an event catalog) and then to model the vulnerability of exposures to those events. This involves explicit relationships between the parameters of the loss-causing event and the resulting extent of damage depending on the characteristics of the exposure. Taking this approach allows examination of the variation in damage across different exposures. Moreover, by varying the characteristics of the exposure, the effects of various mitigation measures can be evaluated.

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[1] (Walters & Morin, 1996)
[2] (Burger, Fitzgerald, White, & Woods, 1996)

By providing a large set of simulated events, catastrophe models give an effective means of exploring the tail probabilities, the frequency and severity of the events that are least likely to have been observed in history. Thousands of events than can reasonably be anticipated but have not been observed can be included in the event catalog.

In working with catastrophe models, one of the benefits is not being constrained to evaluating loss to actual exposures, such as an in-force book of business. Hypothetical or "nominal" exposures can be the subject of modeling, thereby providing a view of anticipated losses for a set of potential exposures.

Throughout the catastrophe modeling process there are opportunities to look at variations that may be seen in event frequency or severity. In other words, sensitivity testing is one of the benefits of working closely with a catastrophe model.

Overall, the catastrophe modeling approach provides a more complete representation of risk, which can help communicate risk in its many dimensions and can support planning for contingencies.

To provide a practical context, we will illustrate the use of catastrophe model results in pricing and underwriting flood insurance.

2.3 Standard Catastrophe Model Output

Catastrophe models are capable of generating a range of different outputs. Some of the key outputs are described in the following sections.

2.3.1 Event Losses

The most basic output of a catastrophe model is the estimate of losses for every simulated event. For most catastrophe models today, these losses can be retained at a range of granularities, allowing the model user to understand the amount of loss estimated for a given simulated event over any subset of a given portfolio, such as by geographic boundaries, lines of business, or even individual policies and locations. All other loss metrics are derivatives of these event loss estimates, and therefore all other metrics are able to be calculated at the same range of granularities available for event loss estimates.

Event loss estimates are frequently referred to as Event Loss Tables (ELT) or Year Loss Tables (YLT), which are simply tables including the losses from each simulated event or year respectively. The Year Loss Table would include the total of the loss estimates for all events that occur in each simulated year.

2.3.2 Average Annual Loss

In a ratemaking context, arguably the most important catastrophe model output is the Average Annual Loss (AAL). The AAL is the expected value of losses to be experienced in any given year. It is equal to the sum of all simulated event losses multiplied by the probability of each of those events. Many catastrophe models assign all simulated events or years the same probability.

2.3.3 Loss Costs

The loss costs are a natural by-product of the AAL. They allow AAL values to be level-set to the amount of exposure that generated the AAL. They are calculated as the AAL divided by a standard exposure metric, such as total insured value.

2.3.4 Exceedance Probability Curves

Exceedance Probability Curves (EP Curves) are the loss output that is most commonly associated with catastrophe models. They represent the probability of exceeding different levels of loss for the catastrophe peril(s) being analyzed. They are sometimes referred to as PML (Probable Maximum Loss) Curves, although this nomenclature can be misleading by implying that the represented loss is the maximum loss that can be expected, when there is almost always some probability of greater losses.

Exceedance probability values are generally expressed in terms of both the probability and the amount of loss. For example, the X% exceedance probability equals $Y. This statement means that there is an X% chance of meeting or exceeding $Y of loss in any given year.

There are two different types of EP Curves that are both common. The Occurrence EP Curve (OEP Curve) represents the probability of the LARGEST event loss in any given year meeting or exceeding different values. The Aggregate EP Curve (AEP Curve), on the other hand, represents the probability of the total losses from ALL events in any given year meeting or exceeding different values.

EP Curves are also frequently expressed in terms of a return period rather than the exceedance probability. The return period is simply the inverse of the exceedance probability. For example, a 1% exceedance probability is equal to a 100-year return period. Similar to the usage of PML, the return-period term can be misleading by implying a period of time that would be expected to pass between events of that magnitude, when in reality they are representative of the probability of meeting or exceeding that level of loss in any given year.

The exceedance probability is also frequently referred to as a Value at Risk (VAR). A VAR of 1% is equal to the 1% exceedance probability.

2.3.5 Standard Deviation

The standard deviation is a common metric in many statistical applications, including catastrophe modeling. Depending on the context, the standard deviation derived from a catastrophe model can represent different things.

If the standard deviation is being referred to in the context of an AAL, EP Curve, ELT, or YLT, it will typically refer to the standard deviation of the distribution of event or annual loss estimates. This is the most common utilization of the metric in catastrophe modeling.

Many catastrophe models incorporate the concept of secondary uncertainty. The loss estimates generated by a catastrophe model represent mean estimates of damage for a given event. In reality, there is uncertainty in the amount of damage that would be experienced from a given event. This uncertainty is represented by a distribution (the secondary uncertainty distribution) that can be persisted through the financial calculations of a catastrophe model, and is an important way to quantify the expected impact of policy conditions when the amount of damage is uncertain. When a standard deviation is being referred to in the context of a single event loss estimate, it refers to the standard deviation of the secondary uncertainty distribution.

2.3.6 Tail Value at Risk

Tail Value at Risk (TVAR) is a derivative of the EP Curves. Rather than representing the probability of exceeding a given level of loss, it represents the average of all expected losses beyond a given exceedance probability. It is equal to the sum of all points on the EP Curve multiplied by the probability of each of those years or events occurring. It represents a conditional expectation of how large losses can be, conditioned on having exceeded a given threshold. It is a common measure of the tail of the EP Curve.

2.3.7 Intensity

While the primary outputs of catastrophe models are estimates of financial loss, an intermediate output can be the intensity of each simulated event at each modeled location. The measures of intensity will vary by the peril being modeled, and are frequently used in the validation of modeled loss output. Wind speed (hurricanes, severe thunderstorms, and winter storms), peak ground acceleration (earthquakes), and flood depths (storm surge and inland flood) are frequently-used intensity metrics, but every model has its own specific set of intensity measures.

2.4 Types of Catastrophe Flood Models

Flood risk is often expressed in terms of flood zones, depicted on maps. That can be a handy way of identifying the level of flood hazard. Typically, a flood hazard map may show the area where it can be expected that water of some specified depth will be present at least once in, say, 100 years, i.e. the 100-year flood zone. In contrast to this approach, a fully probabilistic model will typically use tens of thousands of simulated events to represent the hazard. A hazard map can be a by-product of the model. Going beyond hazard, the model will also reflect the vulnerability of actual or hypothetical exposures to those events and will translate the extent of damage into financial terms with or without the application of insurance coverage terms.

Hurricane or, more generally, tropical cyclone models frequently include the coastal flooding caused by storm surge. Inland flood models allow for evaluation of the impact of events away from the coast. They may be limited to what happens within flood plains, the result of excessive water going beyond the banks of a stream or river, also known as *fluvial* or *riverine* flooding. Inland flooding may also occur outside of flood plains when rain brings more water than a local area can absorb. This latter type of flooding is called *pluvial* flooding.

2.5 Model Validation

The use of catastrophe models has become standard operating procedure in many areas and usage continues to grow as the modeling process evolves and is applied to additional perils. Still, use of models is not central in the work of many actuaries so there is a distinct learning curve involved for many actuaries when it comes to catastrophe modeling. The US Actuarial Standards Board has issued a useful document titled "Using Models Outside the Actuary's Area of Expertise,"[3] which defines the requirements for American actuaries using models that fall outside their area of expertise. Canadian actuarial standards also generically address the use of models in actuarial work, although this advice is spread throughout Section 1000 – General.[4]

Typically, the underpinnings of a catastrophe model fall outside a pricing actuary's area of expertise, as they involve complex representations of the physical characteristics of the peril, as well as the subsequent damage to insured structures. For the flood peril in particular, this would involve experts in climatology, meteorology, hydrology, geographic information systems, and structural engineering – all of which could be outside the actuary's area of expertise. As a result, ASOP 38 requires that actuaries relying on the model do the following:

1. Determine appropriate reliance on experts
2. Have a basic understanding of the model
3. Evaluate whether the model is appropriate for the intended application
4. Determine that appropriate validation has occurred
5. Determine the appropriate use of the model

While Canadian actuaries are not subject to the American Academy of Actuaries' Actuarial Standards of Practice, the requirements laid out by the American Academy provide a solid framework on which to ensure valid model usage.

Actuaries should confirm that appropriate experts from the fields listed above were involved in the development and review of the model. They can review CVs and credentials of the experts, and determine what areas of the model those experts were involved in developing. They should also ensure that appropriate third parties have reviewed the models in addition to the initial developers.

Generally, catastrophe model vendors provide documentation on the development of the models, to help users understand the basic components of the model, as well as the input and output data involved. Actuaries should review this documentation to ensure they understand the different components and how they interact to generate loss estimates.

As part of the evaluation of the model components, actuaries should ensure that the model is generating appropriate output. For catastrophe models, this would include average annual losses for the policies being analyzed, as well as distributions of event losses for the portfolio, or appropriate sub-sets of the portfolio. Average annual losses should reflect the long-term average losses for the policies in the portfolio.

The loss perspective of the model output should correspond to the loss experience being simulated. If direct losses are being simulated, the model should take policy conditions such as limits and deductibles into consideration appropriately. If losses net of reinsurance are being simulated the model should additionally apply reinsurance terms and conditions. If, for any reason, this is not possible, the actuary should make appropriate modifications to the model output to ensure equivalency.

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[3] (US Actuarial Standards Board, 2011) – ASOP 38
[4] (Canadian Actuarial Standards Board, 2017) – Paragraphs 1110.31.1 to 1110.31.5, Subsection 1535 (Models), 1540.01.1, 1540.05 to 1540.09, 1560.09 to 1550.11, 1619.12, Section 1700 (Assumptions), 1820.01, and 1820.26.1 to 1820.26.3

The perils being modeled should be representative of the perils being rated. Non-covered perils should be excluded from model results, and any non-modeled perils should be accounted for separately in any rates. Similarly, if there are any policies in the portfolio being analyzed that are not included in the model results for any reason, whether outside the model domain or for which there is no damageability relationship in the model, those policies should be accounted for separately.

Thorough validation of the model should have occurred, from a number of perspectives. Below are some validations that are common for catastrophe models, but specifics can vary depending on the catastrophe model and what it is intended to represent:

1. Event frequency can be compared to historical event frequency, or scientific consensus on future event frequency.
2. Intensity footprint from modeled historical events can be compared to actual observations of intensity from those events.
3. Distributions of damage from recent events can be compared to modeled loss estimates from the exposure data in force at the time.

Finally, actuaries must ensure appropriate usage of the model output. The loss costs generated by the model must be appropriately brought into ratemaking formulas, and any reinsurance recoveries allocated appropriately to adjust the rates.

Section 3 Current Use of Catastrophe Models in Canadian Ratemaking

3.1 Common Catastrophe Ratemaking Practices in Canada

As part of the background research for this paper, the team conducted a stratified survey with participation from 11 Canadian insurance companies to identify the range of ratemaking approaches currently being used in Canada to incorporate actual catastrophe experience or model output. The survey was conducted through telephone interviews in the first quarter of 2017. The insurers were chosen by the project team and all who were approached agreed to participate in the survey process. Insurance companies were chosen with the objective of ensuring we interviewed:

- Both national and region-based insurers
- Large and mid-market insurers
- Insurers with broker channels and with direct marketing distribution
- Reinsurers as well as direct writers
- Canada-domiciled insurers as well branches/subsidiaries of multinational insurance companies

A copy of the question script used in the survey is provided in Appendix A — Survey Questions for Canadian Insurers.

As agreed with the participants, details of the responses to the survey questions are provided in this report on an anonymous basis only. Individual insurance company responses are not to be made available. The following is a summary of the survey findings.

We discovered a range of catastrophe ratemaking approaches is currently in use, with variances depending on size of insurer, location of risk, and type of peril; more specifically:

- Earthquake risk
    - All participants with earthquake risk exposure indicated that they use the results of a catastrophe simulation model to determine earthquake rating territories (including mapping of excluded regions) and often to develop rates for earthquake coverage.
    - Many participants indicated that reinsurance costs for earthquake coverage are incorporated into property pricing in earthquake-prone regions.
    - Catastrophe model results are used in underwriting and risk management areas as well. The monitoring and management of the concentration of exposure was a key aspect indicated by many.

  - Most participants indicated that pricing typically also reflects loads for additional specific catastrophe expenses, capital costs, and reinsurance costs.

- Approaches to catastrophe ratemaking for perils other than earthquake (wildfire, windstorm, hail, flood) vary among participants; these include the following:
  - Some insurers described an approach whereby results from a third-party or internally developed catastrophe simulation model are incorporated into their ratemaking exercise and underwriting policies. Results of these models are sometimes only used to develop rating areas and zones. In other cases, the annual aggregate loss metrics from the models are developed for each rating area and zone and these are used for developing rates for the specific catastrophe risk.
  - Other insurers described approaches whereby a blend of historical experience is used along with catastrophe simulation model results from third parties. Varied approaches exist depending on the availability of historical loss data by peril and the availability of catastrophe models. When using historical information, catastrophe loss amounts are typically separated from non-catastrophe losses. Participants usually track actual experience from as much credible history as is available and most of them will trend historical claims levels to reflect inflationary increases in claim cost levels.
  - For perils such as wildfire and flood (more specifically sewer back-up coverages), most participants indicated the use of only their own (adjusted) historical experience for ratemaking. The lack of a credible industry catastrophe model is the main reason for this approach.

Additional costs arising from catastrophe exposure are sometimes allocated on a line of business (LOB) or regional basis on top of the insurer's own historical experience and/or industry-based data (where available and where historical information was less credible). There was much variance in how catastrophe exposure costs are allocated in ratemaking approaches. For example, sometimes reinsurance cost loads are applied in aggregate or on a peril-specific basis while at other times reinsurance costs are not explicitly included in the rates.

The majority of participants indicated their ratemaking approach had been in place for several years. However, they all continue to review their processes to refine and enhance their ratemaking approaches on an ongoing basis. These enhancements very often included the development of modeling capabilities or use of external models.

The following comments were received from participants when asked about the issues or problems posed by current practices:

- Lack of credible experience data in certain regions or areas
- Difficulty with assessing new perils or risks
- Model risk and implementation issues (e.g. the lack of internal knowledge of models)
- Inability to predict future perils that are unforeseen or overlooked (e.g. a situation like the 2016 Fort McMurray wildfire)

3.2 Comparison of Canadian and US Flood Insurance Practices

There are generally no regulatory requirements to file property rating plans in most Canadian provinces and specifically no filing requirements for catastrophe rating plans in any Canadian jurisdiction. In the US, rating programs incorporating catastrophe model output are typically the subject of state regulatory filings. Some states, e.g. Florida, even mandate some type of approved catastrophe modeling in pricing hurricane coverage for homeowners' insurance.

Flood insurance has only just begun to be offered through private insurers in Canada. Generally, the burden of flood recovery programs has historically fallen on taxpayers through federal Disaster Financial Assistance Arrangements (DFAA).[5] In the US, the National Flood Insurance Program (NFIP) run by the federal government provides flood insurance for properties in flood-prone areas. It is estimated that nationally only 50% of homes in flood zones are insured by the NFIP.[6]

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[5] (Insurance Bureau of Canada, 2005)
[6] (Dixon, Clancy, Seabury, & Overton, 2006)

3.3 Problems with Current Practices

This section outlines the issues with current ratemaking practices in Canada, identified through the survey of Canadian insurance companies as well as the literature research process.

For insurers who are using ratemaking approaches that focus on historical experience and do not incorporate catastrophe simulation models, the following issues should be considered:

- The lack of appropriate historical data and credible experience
- The difficulty in estimating low-frequency, high-severity events with limited years of history
- A retrospective approach may not account for changes in future circumstances and the related environments affecting potential future experience
- The range of ratemaking practices across the industry may be of concern to those responsible for regulating and monitoring the risk exposure of the market to catastrophic events
- There may be some difficulty in accurately reflecting expenses and costs in rates and allocating such to specific LOBs and/or perils
- The difficulty in capturing tail risk when only relying on a distinct set of historical experience
- The difficulty in pricing new areas and regions which are being considered for expansion
- The difficulty in establishing risk zones and mapping of risks for pricing and underwriting purposes

From our survey research it is clear that pricing methodology should be significantly enhanced by incorporating the use of results from catastrophe simulation models along with historical experience information. This can allow for a more robust approach to ratemaking and management of risk exposure. However, for those who are using catastrophe simulation models to develop rates or in their underwriting processes, either in isolation or combined with historical experience data, the following issues and concerns should be considered:

- A lack of understanding of the model by users
- The development of an appropriate and consistent methodology for incorporating catastrophe simulation models into the ratemaking calculations
- The need to assess whether the model effectively relates to the specific risk exposure of the individual insurance company using it
- The lack of models developed specifically for Canada and the issues with altering a model initially developed for use in other countries or regions (e.g. using a model developed for US wildfire risks in California to project future losses on Canadian wildfire zones)
- The availability of models for new perils
- The costs of model development or purchase

Along with the above concerns, the majority of respondents from our survey of Canadian insurers indicated a desire to have access to tools and models which were more specific to the perils and risks of Canada. They all also indicated that the development of a practical approach to ratemaking which incorporates catastrophe simulation model results was definitely preferred over using historical experience alone.

Section 4 Current State of Flood Insurance in Canada

As you will see in Section 6 below, a proof of concept for the ratemaking approach proposed in this report focuses on the area of flood risk. This section provides details of the current offerings in the Canadian market and the Canadian insurer ratemaking practices specifically in the area of flood risk. This is being provided for flood risk in order to support and provide background for the proof-of-concept example that will be detailed in later sections.

4.1 Standard Policy Offerings

We performed online research into the water damage insurance coverage being offered by eight Canadian insurance companies to provide a sense of the types of products currently being offered in Canada. All of these companies were participants in the survey noted above on ratemaking practices.

In general, all the companies are offering enhanced water damage protection through optional endorsements to policyholders. These endorsements are in addition to coverage offered by the base policies, and all companies are pricing the endorsements separately from the base policy coverage rates. The following are further details regarding the endorsements offered in Canada.

There are generally two approaches used among the products reviewed: one approach where enhanced water coverage is bundled into one endorsement, and a second approach where two separate endorsements are used. The main coverages offered through the endorsements are protection from sewer back-up and overland water protection.

The companies offering two separate endorsements typically separate the sewer back-up from the overland water protection. The product details suggest that this allows for lower premium rates for those with lower exposure to overland water risk; e.g. those that are farther from flood plains. That said, there are companies that also require sewer back-up coverage as a pre-requisite to the purchase of the overland water protection endorsement.

The majority of sewer back-up endorsements include coverage for:

- Backing up of water or sewage from drains, pipes, or fixtures connected to the public sewer system
- Sump pump back-up
- Retention tank or septic tank back-up

The majority of overland water protection typically includes coverage for:

- Water damage caused by the overflowing of a lake/river
- Damage caused by water due to heavy rains or rapid snowmelt that enters a home from a point on or above the ground surface

Exclusions or limiting provisions in the endorsements typically include the following:

- Coverage is not available to property owners in certain highly prone areas or owners of certain properties (mobile homes, seasonal properties, properties with reverse sloped driveways)
- Provisions that eliminate doubling up of claims under the different endorsements, typically separating out what is covered under sewer back-up versus overland water
- Exclusion of flooding caused by ground water or rising of the water table, waves, the tide, tidal waves, a tsunami, a storm surge, saltwater, a dam break, or coastal flooding
- Exclusion of situations with clogged drains or weeping tiles, deterioration or corrosion of roofs, improperly installed downspouts/eaves troughs/vents, or improperly sealed roof flashings or vent flashings

There are also some endorsements which offer additional protection for specific circumstances causing ground water to enter the home which are not covered in the basic homeowner's insurance policy.

It is also indicated by several companies that endorsement coverage, limitations, and exclusions may also vary by province.

The recent addition of flood endorsements to the market comes as a result of water-related claims becoming the leading cause of home insurance losses across the country. Despite the lack of coverage under base policies in the past, insurers often ended up paying for flood-related damage in the event of major floods, due to difficulty in ascertaining the causes of loss and a desire to avoid reputational damage during major flood disasters.

It is also important to note that, as indicated above, in the past much of the financial burden related to flood disasters in Canada has fallen on taxpayers through disaster relief programs provided through municipal and provincial governments with financial support at times from the federal DFAA program. In the event of large-scale natural disasters, the Canadian federal government provides financial assistance directly to provincial and territorial governments through the DFAA program. Assistance payments from the federal government are triggered when response and recovery costs of natural disasters exceed what individual provinces and territories could reasonably be expected to bear on their own. This trigger point varies by province, based on population. The DFAA provides increasing levels of support as the cost of disaster recovery rises.

However, the costs of restoring or replacing items that were insured or insurable are not eligible for coverage under the DFAA: “Under the DFAA insurable means that insurance coverage for a specific hazard for the individual, family, small business owner, or farmer was available in the area at reasonable cost. Reasonable cost and availability are determined jointly by the province and the Public Safety Canada RD [Regional Director].”[7] Eligible costs to a province under the DFAA are net costs after any recoveries from insurance payouts (among other recovery sources).

With the new endorsements outlined above being offered in the marketplace, insurance companies have now taken on a larger portion of liability for flood risk as the first payers of flood-damage-related costs for policyholders who live in areas that are insurable in Canada. Government disaster relief programs are designed to pick up costs for losses above insurance losses where applicable. That said, provincial relief programs and the DFAA are still going to be subject to losses occurring in areas that are uninsurable due to being close to flood plains and highly prone to flood damage. In these areas, coverage is often unavailable.

4.2 Current Rating Methodology

The methodologies used by Canadian insurers for pricing the previously described enhanced water endorsements do vary. However, there are several common characteristics among them as well.

Water endorsements are priced separately from the base property policy coverages, with the exception of some commercial policies. Some companies include the additional water coverage as part of their basic comprehensive commercial policies.

The information below regarding ratemaking practices for flood endorsements is taken from our previously described survey of Canadian insurers’ catastrophe ratemaking practices, the results of which are also detailed in Section 3 above.

- It is common for insurers to use their own historical experience along with the results from a third-party catastrophe simulation model in pricing for additional water damage coverage, as well as in the underwriting of these risks.
- Some insurers are not explicitly using catastrophe model results in their rating methodology. However, the majority are using these models for territorial mapping of pricing zones and to establish underwriting policies.
- It appears that the lack of any Canada-specific models and the fairly recent development of these flood endorsement products for the Canadian market are the key reasons for the slower development of ratemaking practices using catastrophe simulation models.
- Some insurers also indicated that their rating methods are always in development and enhancement mode and that catastrophe model development is a key area in which the flood pricing enhancement may occur.
- Ratemaking approaches reflect the need to accurately account for additional expenses and cost from catastrophe policy risks, which include reinsurance costs and costs of capital incurred from reinsurers.

Section 5 Proposed Ratemaking Approach

The following section will describe, in general terms, the approach recommended by the research team for incorporating catastrophe modeling output into insurance rates for the flood peril in Canada. Many of the principles will be broadly applicable to other catastrophe perils as well, but some considerations will be specific to flood.

5.1 General Ratemaking Concepts

The sections below will lay out a general approach for incorporating catastrophe models into ratemaking. Some aspects are relatively standard across ratemaking, but some aspects are more specific to catastrophe perils, such as risk loads and reinsurance expenses.

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[7] (Public Safety Canada, 2007)

5.1.1 Pure Premium Approach

A standard approach to ratemaking is the pure premium approach.[8] In this approach, the pure premium represents the average premium per exposure unit. The average rate incorporates average losses and Loss Adjustment Expense (LAE), a risk load, expenses, and profit provisions, according to the formula below.

$$Average\ Rate = \frac{Average\ Loss\ and\ LAE\ Pure\ Premium\ +\ Risk\ Load\ +\ Fixed\ Expense\ Per\ Exposure}{1 - Variable\ Expense\ and\ Profit\ Ratios}$$

Generally, the average losses would be taken from historical loss information for the insurer, but for the purposes of a catastrophe peril such as flood there are limitations in the usage of actual loss experience. These limitations stem from the fact that catastrophe events are infrequent and difficult to predict, and can generate highly correlated losses to a set of exposures when they occur.

The pure premium approach is therefore modified in the case of catastrophe-prone perils to replace all or part of the historical loss experience with modeled loss estimates. Due to the very limited history of flood insurance in Canada, there is limited historical loss experience in the industry to begin with. In some cases, insurers have been looking at historic sewer back-up claim experience to supplement sparse flood claim experience. As a result, in this case, utilizing modeled loss estimates as a replacement for any historical loss experience is preferable. So, referring to the formula above, “Average Loss” would be defined to reflect the AAL produced by a catastrophe model.

An alternative to the pure premium approach would be the loss ratio approach. The pure premium approach is more applicable to generating a rating plan for a new line of business or peril, while the loss ratio approach is more appropriate for calculating indicated rate changes. Since this paper is evaluating a new peril, the pure premium approach was selected, but the use of the loss ratio approach is theoretically identical, and loss experience from a model can similarly replace part or all the loss experience that would traditionally be used in the loss ratio approach.

5.1.2 Risk Load

In addition to the averages losses, the average rate formula above calls for a risk load reflecting the cost of taking on the risk for the insurer. This is especially important in the case of catastrophe perils, where the expected volatility in the loss experience is much higher than traditional insured perils like fire or theft. Kreps lays out a framework for calculating the risk load using a reluctance factor, according to the following formula:[9]

$$Average\ Risk\ Load = \frac{R * \sigma}{Exposure}$$

Where R = Reluctance Factor calculated according to the Kreps formula and σ = the standard deviation of the loss experience. The formula to define R is as follows:

$$R = \frac{y * Z}{1 + y}$$

Where y = the Expected Return and Z = a selected point on the normal distribution reflecting the desired percentile of risk (typically 95th or 99th percentile). Again, the loss distributions utilized to generate the standard deviation are best determined using a catastrophe model.

The theory behind the selection of a reluctance factor according to Kreps begins with the acknowledgement that surplus is required to support the variability of portfolio experience. Kreps represents the required surplus as equal to a distributional point corresponding to an acceptable probability (Z) that the actual result will require more surplus (S) allocated times the surplus minus the expected return (r). This is represented by the following formula:

[8] (Werner & Modlin, 2009) – Chapter 8
[9] (Kreps, 1990)

$$S = Z * \sigma - r$$

The expected return is also equal to the expected portfolio return on surplus represented as:

$$r = y * S$$

As both these are definitions of the expected return, substituting results in:

$$y * S = Z * \sigma - S, \text{ or}$$

$$S = \frac{Z * \sigma}{(1 + y)}$$

Substituting in the expected return formula ($r = y * S$) from above:

$$r = \left(\frac{y}{1 + y}\right) * Z * \sigma$$

Since the risk load is defined as the reluctance factor (R) multiplied by the standard deviation (σ), the resulting formula is as follows:

$$R = \frac{y}{(1 + y)} * Z$$

Consequently, the primary considerations for the practitioner are the selection of Z equal to a distribution percentage point corresponding to an acceptable probability that the actual result will require more surplus allocated and the y equal to the desired portfolio yield. These selections will likely be affected by the actuary's view of market conditions, risk aversion, and the company's capacity.

The risk load methodology described herein, using the standard deviation, is only one method for calculating the risk load. It is the most common in practice. Alternatives include using VAR, TVAR, and other more complex methods that leverage simulation techniques. There is a range of actuarial literature describing and evaluating these methods, including "Catastrophe Pricing: Making Sense of the Alternatives" by Ira Robbin[10] and "An Introduction to Risk Measures for Actuarial Applications" by Mary Hardy.[11] Evaluating the relative merits of these methodologies will be discussed briefly in Section 7 of this paper. However, it should be noted that the use of a catastrophe model in lieu of historical data would improve the accuracy and stability of these metrics in much the same way that it does when leveraging the Kreps method.

5.1.3 Other Rate Components

Beyond the loss-based rate components, there are several expense and profit provisions incorporated in the formula above. The provisions include the following:

1. Commissions
2. Taxes
3. Fixed Expenses
4. Underwriting Profit Provision (Profit)
5. Investment Return
6. Premium to Surplus Ratio (PS)
7. Trend Factor
8. LAE

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[10] (Robbin, 2013)
[11] (Hardy, 2016)

The determination of these various factors is outside the scope of this paper, but their consideration is important to the calculation of the overall rate. Incorporation of these factors results in the following formula for the average rate:

$$Average\ Rate = \frac{[(Average\ Losses) * (1 + Trend) * (1 + LAE)] + (Risk\ Load) + (Fixed\ Expense)}{\{1 - Commisions - Taxes - \ Profit + [(Investment\ Return) * (1 + \langle 1/PS \rangle)]\}}$$

5.1.4 Net Cost of Reinsurance

Since reinsurance is a frequent risk transfer mechanism for catastrophe perils, it is necessary to incorporate the cost of, and recoveries from, that reinsurance in determining the premium to be charged. Adjusting the perspective of the calculation to net of reinsurance involves adding the cost of reinsurance as a fixed expense and adjusting the risk load and average losses to reflect recoveries from reinsurance. This modifies the formula laid out above as follows:

$$Average\ Rate = \frac{[(Average\ Net\ Losses\ ) * (1 + Trend) * (1 + LAE)] + (Net\ Loss\ Risk\ Load) + (Fixed\ Expense) + \ (Reinsurance\ Cost)}{\{1 - Commisions - Taxes - \ Profit + [(Investment\ Return) * (1 + \langle 1/PS \rangle)]\}}$$

Once again, catastrophe models can be used to estimate the impact of reinsurance on the expected losses. These impacts can then flow through the calculation of the standard deviation in the risk load calculation. The cost of reinsurance would be determined based on market conditions in the reinsurance market, but can be estimated for a particular portfolio using similar techniques to what is described above, but for the expected losses to the reinsurance treaty.

For the purposes of this paper, we have assumed the reinsurance is loaded as a flat expense, but in some cases it may be preferable to allocate the reinsurance premium based on the losses expected from different risks within the portfolio. Catastrophe model output can be readily used for this purpose as well, i.e. reinsurance could be allocated to territory based on territorial losses, risk loads, or components of the risk load, such as the standard deviation.

5.1.5 Relativity Factors

Thus far, the techniques described have been appropriate for determining the average rate for the portfolio, but with any insurance pricing plan it is necessary to differentiate between risks to ensure you appropriately reflect the relative risk of different policies and avoid issues of adverse selection. This differentiation is frequently determined through the use of relativity factors, adjusting the average rate to reflect the reduced or heightened risk of a specific policy.

The goal with relativity factors is to determine the appropriate rate differential between different risks. These factors are generally multiplicative against the base rate, and are calculated by comparing the expected losses from a base risk to the expected loss for the exposure characteristic being evaluated. Traditionally, historical loss information can be used to calculate these expected loss differentials, but as with prior descriptions of loss experience, it is more robust to use a model for expected catastrophe losses, and therefore rate differentials.

The specific relativity factors that would be incorporated will vary based on the nature of the final rating plan being developed, but at a high level would include the following types of factors for property rating:

1. Exposure Factor: used to adjust the average rate for the difference in the exposure base between the policy being rated and the average policy
2. Territory Relativity: used to adjust the average rate based on the expected level of loss in the territory to which the policy being rated belongs
3. Policy Relativity Factors: used to adjust the average rate based on relative riskiness of different policy attributes, such as excess limits or deductibles
4. Building Characteristics Factors: used to adjust the average rate based on relative riskiness of different building characteristics, such as construction, occupancy, year built, or mitigation features

More detail on this process is provided in Section 5.2 below, which describes a methodology to go beyond modeled catastrophe losses for a portfolio to eliminate any bias inherent in the exposure data in the portfolio.

5.1.6 Off-Balancing Average Rate to Portfolio

At this stage, there is an average rate for exposures in the portfolio and relativity factors that are used to adjust that average rate. The next step would be to calculate the rate for each individual policy within the portfolio and off-balance those rates to ensure the average rate for the portfolio is maintained.[12]

Each policy's rate is determined by multiplying the average rate by all relevant relativity factors for that policy. The result is the calculated rate that will be charged for each individual policy in the portfolio. The average of these rates is then determined (the Average Calculated Rate) and compared to the Initial Average Rate. The ratio of the two values is then applied to the Initial Average Rate to calculate the base rate used in policy rate determination:

$$Final\ Base\ Rate = \ Initial\ Average\ Rate * \left(\frac{Initial\ Average\ Rate}{Average\ Calculated\ Rate}\right)$$

The individual policy rates are then recalculated by multiplying the final base rate by the relativity factors for each policy, and the average of those will be equal to the Initial Average Rate. This ensures that the overall expected loss for the portfolio is maintained once all relativity factors have been incorporated.

5.2 Use of Notional Exposure Data

Modeled losses for the actual portfolio are very valuable for providing the long-term view of average losses, but the models can be even more valuable in their ability to eliminate any exposure bias in the calculation of relativity factors using a hypothetical set of policies known as notional exposure data. Catastrophe models can analyze any property data record for which sufficient information is provided regarding the property's location, structural attributes, and values. It is not necessary that the record is identifiable as an actual existing structure. A notional data set is comprised of hypothetical properties with a uniform set of base characteristics, which can be repeated with certain variables modified to evaluate their impacts. The key advantage of this approach is the ability to capture all geographical areas and structure types to be analyzed without gaps or concentrations in certain common structure types.

This approach can be used for both the determination of territory definitions and rating relativity factors. The specifics for each of these use cases is laid out below. It should be noted that when a model is used for this purpose it must be validated that the model itself has no bias and does indeed replicate the impact of the underwriting criteria being used to allocate risks.

5.2.1 Territorial Analysis

The goal in setting territories for a rating plan is to group geographic areas with similar expected loss potential. This is particularly important with catastrophe perils, where there is significant geographic correlation in loss potential. Using historical loss experience for this process is going to significantly distort loss expectations; increasing relativities in areas with recent catastrophe experience and greatly reducing relativities in other areas. Thus, capturing modeled loss estimates in territorial relativities provides a more stable long-term view.

However, it is common to see significant differences in the makeup of the insurance portfolio when comparing more catastrophe-prone areas to less risky areas. For example, the average building in a flood-prone region is built with resilience to flood in mind, while buildings in less exposed areas are not. Most catastrophe models should account for different construction types and mitigation features, and would reflect the different loss expectation for those building characteristics. The same is true for differences in the average policy conditions. For example, it is common to have a separate (higher) deductible, or a lower limit for flood risk, within the 100-year flood hazard area, and the loss estimates generated by the model for a portfolio in that area would reflect that. If you were to define territories based solely on the modeled loss estimates for the actual portfolio, you would be muting the territorial relativity with these loss mitigation factors. This problem is exacerbated if there are separate relativity factors applied within the rating plan that are intended to capture the effects of these mitigations.

[12] (Werner & Modlin, 2009) – Chapter 14

To reduce the bias inherent in the makeup of the actual portfolio, it is possible to create a notional portfolio that places a hypothetical location record at each geographic location within the actual insured portfolio. Each hypothetical location record will have the same replacement values, policy conditions, and building characteristics (defined as the base risk), so the only variability being captured in the modeled loss output is driven by differences in hazard within the geographies being examined.

By maintaining the geographic distribution of the existing portfolio, the rating plan will be assured of capturing the areas where business is being written, and providing more robust estimates in areas of highest risk concentration. By leveraging the model for this process, you are able to entirely eliminate exposure biases in the model in addition to eliminating gaps in the historical loss experience from low-frequency catastrophe events.

The catastrophe model will generate an AAL for each location record. It is then possible to apply geographic groupings to those location records to minimize error in the territory definitions. Traditionally, this would involve leveraging combinations of geographic information such as geopolitical boundaries (postal codes, Forward Sorting Areas [FSAs], municipality boundaries, or provinces), geographic landmarks (rivers, roads, lakes, etc.), or areas of particular hazard (flood hazard zones, soil conditions, distance to coast).

When grouping locations into territorial definitions, it is important to consider the credibility of the loss data within each territory. This involves determining that each territory has sufficient loss records to appropriately reflect the average annual expected losses within the territory. Use of a catastrophe model further helps in this regard by replacing a relatively short historical loss experience period with tens or even hundreds of thousands of simulated years of loss activity. It is still preferable to ensure multiple location records are contained within each territory definition to ensure that the hazard conditions at a single geographic point are not having undue influence on the rates to be charged for future policies within that territory.

Statistical tests can be applied to the average relativities of these territory groupings to determine the appropriateness of the territory definitions. The F statistic can be used for this purpose. The F statistic may be interpreted as the ratio of between-class variance to within-class variance. The reciprocal of this is the ratio of within- to between-group variance. This latter statistic is a commonly employed measure in the evaluation of territories and other classification systems, and shows that the territorial definitions are correctly applied when the value is as close to zero as possible for the territory definitions. The average absolute error and correlations coefficients are additional measures that can be useful in evaluating the accuracy of alternate territorial configurations.

5.2.2 Relativity Analysis

The process of setting relativity factors is intended to isolate differences in the relative risk of different policy characteristics. The factors themselves would be multiplied against the base risk to determine the final rate for each policy. As mentioned above, the following factors can all be part of a standard rating plan:

1. Exposure Factor: used to adjust the average rate for the difference in the exposure base between the policy being rated and the average policy
2. Territory Relativity: used to adjust the average rate based on the expected level of loss in the territory to which the policy being rated belongs
3. Policy Relativity Factors: used to adjust the average rate based on relative riskiness of different policy attributes, such as excess limits or deductibles
4. Building Characteristics Factors: used to adjust the average rate based on relative riskiness of different building characteristics, such as construction, occupancy, year built, or mitigation features

Exposure factors are simply used to gross the base rate up or down to the amount of exposure in the policy being considered, and territory relativity factors are addressed in Section 5.2.1 above. For this section, Policy Relativity Factors and Building Characteristics Factors will be addressed, with a similar procedure used for both these categories. For simplicity, both categories are referred to herein as simply “relativity factors.”

In the process of setting territory relativities, it is necessary to remove any inherent bias in the property and policy attributes contained within the portfolio to isolate geographic differences in hazard. When determining appropriate relativity factors, the opposite is true – it is important to eliminate any geographic bias in the portfolio to ensure that the differentials between two attribute values are reflective of the difference in risk solely due to those attribute values, and not any residual effects from locations subject to different levels of hazard.

For example, if you were to examine a portfolio with both coastal and inland exposures for the hurricane peril, you would find a higher concentration of hurricane shutters in coastal areas, and a lower concentration in inland areas. If you simply compared the average loss cost of policies with hurricane shutters to those without, you would likely see that the policies with hurricane shutters had higher average losses than those without, even though hurricane shutters should serve to reduce hurricane damage. While this effect can be mitigated somewhat by isolating the calculation of differentials within each territory, there is still some range of hazard levels within each territory, which would skew the relativity calculation for a given attribute.

Just as it is possible to isolate the territorial relativities by modeling a notional portfolio with constant building and policy characteristics, it is possible to isolate relativity factors by placing multiple notional policy records at each geographic location within the insured portfolio, and systematically varying the building and policy characteristics defined on each policy record. This causes the model to estimate the same intensity for all the location records, ensuring that any differentials in average losses are solely reflective of differences in the building and policy characteristics.

When the various building and policy characteristics are known to be independent of one another within a catastrophe model, it is possible to evaluate the relativity factors for those characteristics in isolation. Deductible relativities could be considered independent of construction factors, for example. However, in many cases, there are intentional dependencies built into the model that would require the different characteristics to be considered in combination with one another. For example, a flood model may make assumptions for the first-floor height of a structure based on the foundation type of the same structure. If you were to provide a credit for having an elevated first-floor height, and a separate credit for having a pile foundation type (which would typically require an elevated first floor), you could effectively be double-counting the impact of that elevation by considering the two in isolation. In these cases, it would be necessary to consider all possible combinations of the different building or policy conditions which are correlated.

The actuary should make sure to understand these interactions within the model to ensure they are structuring the notional portfolio appropriately. By ensuring the appropriate structure of the notional portfolio, accounting for any inter-dependence of building and policy characteristics, and performing the analysis using a catastrophe model, it effectively eliminates the need to perform a minimum bias procedure, which would otherwise be necessary when using historical data to determine relativities. This is a significant benefit for the final rating plan, as any minimum bias procedure would minimize bias, while the use of the model in this context can effectively eliminate it entirely.

By maintaining the geographic distribution of the existing portfolio, the rating plan will be assured of capturing the areas where business is being written, and providing more robust estimates in areas of highest risk concentration. However, when a wide range of policy and building characteristics are being examined, and especially when many of these characteristics are inter-dependent within the model, modeling every possible combination of exposure characteristics at every geographic location within a large insurance portfolio would create a volume of data that may present computational challenges if every location in the insurers portfolio is included. In these cases, it would be appropriate to reduce the number of geographic locations to a more manageable analysis size. This should be done with consideration for the distribution of the expected portfolio and the range of potential hazard levels in the area being rated.

Similar to territorial analysis, it is important to ensure credibility in the loss estimates, and this can be achieved by ensuring that the geographic locations in the notional portfolio are sufficiently representative of the territories within which the relativity factors are being calculated, as mentioned above.

Due to non-linear relationships between hazard and damage, it is possible in many cases that different relativity factors should be calculated for different regions with significant changes in hazard. Statistical tests can again be used to determine if the groupings of relativity factors are appropriate. The reciprocal of the F statistic is a commonly employed measure in the evaluation of relativity factors and other classification systems, and shows that the relativity factors are correctly structured when the value is as close to zero as possible.

5.3 Specific Considerations for the Flood Peril

Most of the methodology described above is broadly applicable to all catastrophe perils, but there are some specifics that should be considered when evaluating the flood peril in particular.

5.3.1 Underwriting Guidelines

Many of the highest-risk properties for flood are going to be subject to very frequent flooding, due to their location within a low-return-period flood plain, so it is especially important to ensure that the highest-risk policies are uncovered in the underwriting process. For example, the NFIP in the United States has estimated that 30% of its claims arise from less than 1% of its policies.[13] Due to this concentration of risk in a relatively small set of properties, it is helpful to have a range of simple criteria with which to avoid adverse selection in the underwriting process.

There are many relatively simple metrics that can be brought to bear in making underwriting decisions, especially for the flood peril. Underwriting criteria specific for the flood peril may include the following characteristics:

1. Return-period flood hazard zones
2. Estimated flood depth within return-period flood zones
3. Proximity to a defined flood hazard zone
4. Elevation
5. Relative elevation
6. Slope of the surrounding area
7. Distance to water bodies
8. Historical precipitation averages
9. Flood protection measures (levees, spillways, etc.) in the area
10. Flood-damage mitigation of the structure

These criteria can be used in combination with a well-structured rating plan to ensure risk selection is coordinated across the portfolio. Risk selection can be a very important part of maintaining a viable flood insurance portfolio.

5.3.2 Territory Definitions

For flood in particular, it is common to see large variations in risk within very small areas, due to the very localized nature of the peril, and its sensitivity to small changes in elevation and proximity to water bodies. Because of this, many traditional geographic boundaries utilized in ratemaking can be difficult to apply to the flood peril, as they cover broader geographic areas that may contain a wide range of flood risk. Thus, the use of fully contiguous territory boundaries can be quite challenging.

In order to overcome this limitation, the adoption of non-contiguous territories can have significant benefit. In this method, rather than grouping the modeled locations within contiguous bounds, the modeled locations are instead grouped simply by having similar loss costs. The territories are thus defined by collections of locations that have very similar risk profiles, rather than contiguous geographic boundaries. This has the effect of greatly reducing variation of estimated losses within a territory. With the improved computing capacity in this day and age, the computational requirements of highly detailed territory definitions like this are much more easily resolved.

In Section 6 below, the proof of concept will demonstrate this non-contiguous methodology for deriving territories, as well as the more traditional approach described in Section 5.2.1 above.

Regardless of the territorial definition approach used, accurate geocoding is crucial in ensuring the rates appropriately reflect the risk of a given property. Due to the highly-location-sensitive nature of flood risk, small errors in the geocoding can have significant impacts on the estimated risk of a property.

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[13] (Smith, 2017)

5.3.3 Rating Variables

In general, the primary rating variables for catastrophic peril ratemaking are the same as those for other homeowners' perils, e.g. construction type, occupancy, and deductibles, but there are specific considerations for the flood peril.

Virtually all damage from flooding comes in basements and the first floor of a structure, so features related to basements and elevation of the building or key components can have a significant impact on loss estimates. Examples include:

1. First-floor height above ground level
2. Elevation of service equipment (e.g. HVAC and electrical systems)
3. Foundation type
4. Basement finish
5. Floor of interest

Section 6 Proof of Concept

6.1 Overview of Sample Company

In order to demonstrate the feasibility of the methodology described within this paper, a sample company will be examined. The two sections below describe the details of that sample company.

6.1.1 Sample Company Portfolio

The sample company has a portfolio of ~20,000 homeowners' policies in the greater Toronto area, as shown in Figure 1 below.

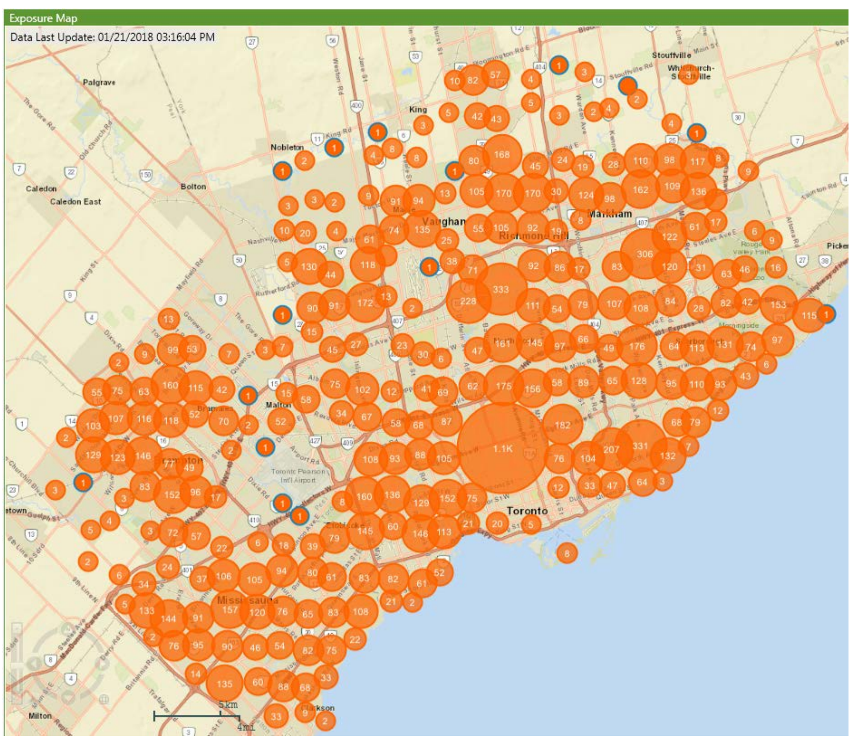

Figure 1: Sample company geographic distribution of policies

The sample company is not intended to reflect any actual insurer, but the distribution of policies, replacement values, construction types, and deductibles is intended to reflect a typical insurer in this area.

The company offers a flood endorsement on its existing policies, and is pricing that endorsement. The endorsement is written to cover losses from both overland flooding and sewer back-up. The endorsement as a standard offering includes the same coverage limits and deductibles as the base policy to which it attaches. Due to the high-risk nature of the flood peril in select locations, this endorsement is offered only on policies that fall outside the 100-year flood plain.

The remaining portfolio of eligible policies consists of 19,206 individual homeowners' policies. These policies have an average building value of $350,843. All policies are written at 100% limit to replacement value. The company has decided to extend this endorsement using the existing policy limits and deductible values. The standard policy form is laid out in Table 1 below.

Table 1: Sample Company Standard All-Perils Policy Form

Policy Term	Value
Building Limit (Coverage A)	Replacement Value of the Primary Building
Appurtenant Structures Limit (Coverage B)	5% of Coverage A
Contents Value (Coverage C)	70% of Coverage A
Additional Living Expense (Coverage D)	20% of Coverage A
Estimated Additional Living Expense Value	$175 per day
Deductible Options	$500, $750, or $1,000 – applied to combined loss from Coverages A, B, and C

Policies are offered in increments of $1,000 of the Coverage A limit. The policy limits in place are distributed as seen in Table 2 below.

Table 2: Sample Company Coverage A Limit Profile

Coverage A Limit	# of Policies	% of Total
$200,000 or Less	56	0.3%
$201,000 to $300,000	6,918	36.0%
$301,000 to $400,000	7,797	40.6%
$401,000 to $500,000	2,773	14.4%
$501,000 to $600,000	1,084	5.6%
$601,000 to $700,000	362	1.9%
$701,000 to $800,000	132	0.7%
$801,000 to $900,000	41	0.2%
$901,000 to $1,000,000	30	0.2%
Greater than $1,000,000	13	0.1%

The portfolio consists of a mix of construction types and deductible values, as shown in Figure 2 and Figure 3 below.

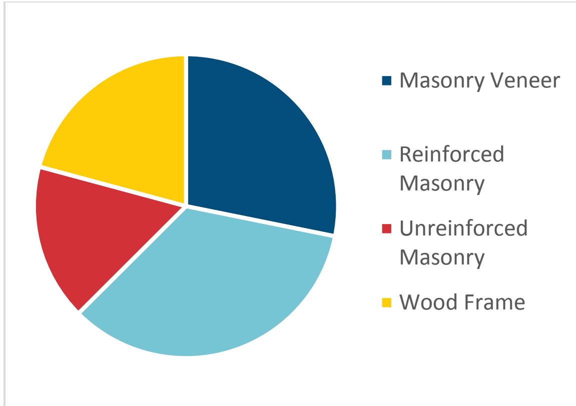

Figure 2: Sample company construction type distribution

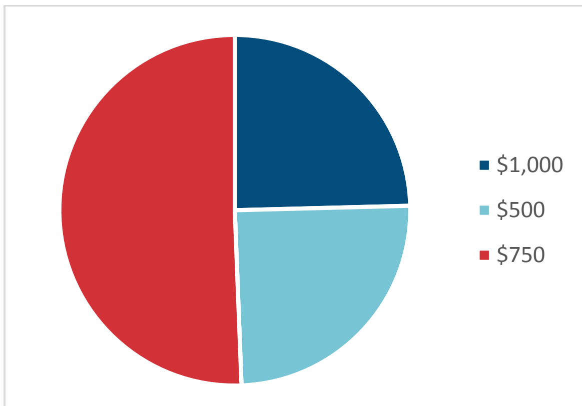

Figure 3: Sample company deductible value distribution

The sample company has purchased aggregate excess of loss reinsurance coverage with a single layer of $35M in excess of an attachment point of $35M. As seen in Table 3 below, this corresponds to roughly the 50- to 500-year return periods. This is a fairly conservative selection, due to the relatively low cost associated with reinsurance outside the flood zone.

6.1.2 Sample Company Loss Estimates

The sample company's portfolio generated loss estimates from a flood catastrophe model as laid out in Table 3 below.

Table 3: Sample Company Modeled Loss Estimates

Loss Metric	Direct ($)	Ceded to Reinsurance ($)	Net of Reinsurance ($)
Average Annual Loss	5,435,547	258,853	5,176,694
Standard Deviation	9,027,140	2,402,193	7,616,193
10-Year Return Period	14,616,732	0	14,616,732
50-Year Return Period	33,906,367	0	33,908,367
100-Year Return Period	45,448,618	10,448,618	35,000,000
500-Year Return Period	71,831,292	35,000,000	36,831,292
1,000-Year Return Period	81,594,727	35,000,000	46,594,727

The primary methodology of this paper is assuming the flood endorsement is being offered for the first time, so there is no actual historical experience to compare to. However, to illustrate the benefits of a catastrophe model over the use of historical claims experience, it was assumed that the sample company's portfolio had loss experience as shown in Table 4 below.

Table 4: Sample Company Historical Loss Experience

Year	Number of Claims	Direct Losses ($)
2003	7	375,487
2004	103	8,169,837
2005	97	8,150,913
2006	49	4,103,903
2007	29	1,952,638
2008	150	13,069,088
2009	33	2,588,974
2010	10	514,923
2011	74	5,585,134
2012	32	1,991,075
2013	15	933,819
2014	129	9,634,851
2015	2	340,155
2016	76	5,026,550
2017	390	33,756,400
Average	**79.9**	**6,412,916**

As the direct losses have never exceeded the attachment point for the reinsurance program, none of the losses have been ceded to reinsurance.

6.2 Applying the Proposed Ratemaking Framework

The following sections will apply through the methodology described in Section 5 above to the sample company. It should be emphasized the results contained herein are only intended to demonstrate the feasibility of the methodology, and do not reflect the opinions of the authors, their respective companies, or any of the sponsoring organizations as to the expected losses from the flood peril. They are not representative of any actual insurance company, and should be used for illustrative purposes only.

6.2.1 Direct Average Rate

As laid out in Section 5.1.4 above, the formula for the average rate prior to the application of reinsurance is as follows:

$$Average\ Rate = \frac{[(Average\ Losses) * (1 + Trend) * (1 + LAE)] + (Risk\ Load) + (Fixed\ Expense)}{\{1 - Commisions - Taxes - \ Profit + [(Investment\ Return) * (1 + \langle P/S \rangle^{-1})]\}}$$

Trend, LAE, Fixed Expense, Commissions, Taxes, Profit, Investment Return, and the P/S ratio are all selected to represent the sample company as those factors are not specific to rating for catastrophe perils such as flood, and calculation of the actual values is outside the scope of this paper. The inputs that are required are therefore the Average Losses and the Risk Load.

The calculation of the risk load is described in Section 5.1.2 above. The formulas used are:

$$Average\ Risk\ Load = \frac{R * \sigma}{Exposure}$$

$$R = \frac{y * Z}{1 + y}$$

The calculations as they relate to the sample company are included in Table 5 below. The exposure base used for the rating methodology is house years.

Table 5: Sample Company Direct Risk Load Calculation

Item #	Component	Value	Notes
(1)	Expected Return (y)	10%	Selected
(2)	Percentile	95^{th}	Selected
(3)	Point on Normal Distribution (z)	1.645	Derived from (2)
(4)	Reluctance Factor (R)	0.14955	= (1) * (3) / [1 + (1)]
(5)	Selected Reluctance Factor	0.15	Selected
(6)	Average Annual Loss	5,435,547	Derived from Model
(7)	Standard Deviation	9,027,140	Derived from Model
(8)	Risk Load	1,354,071	= (4) * (7)
(9)	House Years	19,206	Derived from Sample Portfolio
(10)	Pure Premium	283.01	= (6) / (9)
(11)	**Average Risk Load**	**70.50**	**= (8) / (9)**

The remaining values required for the Direct Average Rate are included in Table 6 below.

Table 6: Sample Company Direct Average Rate Calculations

Item #	Component	Value	Notes
(1)	Commissions	20%	Selected
(2)	Premium Tax	4%	Selected
(3)	Fixed Expense	$25	Selected
(4)	Trend	2%	Selected
(5)	Trend Length (Years)	2.5	Selected
(6)	Investment Return	2%	Selected
(7)	Premium to Surplus Ratio	2:1	Selected
(8)	Underwriting Profit Provision	5%	Selected
(9)	LAE	10%	Selected
(10)	Average Annual Direct Loss	5,435,490	Derived from Model
(11)	House Years	19,206	Derived from Sample Portfolio
(12)	Direct Loss Pure Premium	283.01	= (10) / (11)
(13)	Trended Direct Loss Pure Premium	297.37	= (12) * [1 + (4)] ^ (5)
(14)	Direct Loss and LAE Pure Premium	327.11	= (13) * [1 + (9)]
(15)	Risk Load	70.50	See Table 5

This results in the final resolution of the average rate formula as:

$$Average\ Rate = \frac{[(283.01) * (1.02^{2.5}) * (1.1)] + (70.5) + (25)}{\{1 - 0.2 - 0.04 - \ 0.05 + [(0.02) * (1 + \langle 1/2 \rangle)]\}} = \frac{(327.11) + (70.5) + (25)}{(0.74)} = \ \$571.10$$

6.2.2 Average Rate with Reinsurance

As described in Section 5.1.4 above, the formula for the average rate net of reinsurance is as follows:

$$Average\ Rate\ Net\ of\ Reinsurance = \frac{[(Average\ Net\ Losses) * (1 + Trend) * (1 + LAE)] + (Net\ Loss\ Risk\ Load) + (Fixed\ Expense) + (Reinsurance\ Cost)}{\{1 - Commisions - Taxes - Profit + [(Investment\ Return) * (1 + \langle 1/PS \rangle)]\}}$$

For the purposes of this exercise, we'll be assuming the reinsurance cost based on a similar methodology as laid out for the primary insurance company. In reality, reinsurance cost would be determined by the actual price set within the reinsurance market.

Since the risk profile is going to be altered by the purchase of reinsurance, the risk load also needs to be altered. These calculations are laid out in Table 7 below.

Table 7: Sample Company Risk Load Calculation with Reinsurance

Item #	Component	Direct Basis	Ceded to Reinsurance	Net of Reinsurance	Notes
(1)	Expected Return (y)	10%	15%	10%	Selected
(2)	Percentile	95th	99th	95th	Selected
(3)	Point on Normal Distribution (z)	1.645	2.330	1.645	Derived from (2)
(4)	Reluctance Factor (R)	0.14955	0.30391	0.14955	= (1) * (3) / [1 + (1)]
(5)	Selected Reluctance Factor	0.15	0.30	0.15	Selected
(6)	Average Annual Loss	5,435,547	258,853	5,176,694	Derived from Model
(7)	Standard Deviation	9,027,140	2,402,193	7,616,193	Derived from Model
(8)	Risk Load	1,354,071	720,658	1,142,429	= (4) * (7)
(9)	House Years	19,206	19,206	19,206	Derived from Sample Portfolio
(10)	Pure Premium	283.01	13.48	269.54	= (6) / (9)
(11)	**Average Risk Load**	**70.50**	**37.52**	**59.48**	**= (8) / (9)**

The reluctance factor for the reinsurer assumed a higher expected return and level of confidence to reflect the increased overall risk of reinsurance. Utilizing the same methodology as above, the cost of the reinsurance would be determined using this formula:

$$Average\ Reinsurance\ Rate = \frac{[(Avg.Ceded\ Losses) * (1 + Trend) * (1 + LAE)] + (Ceded\ Risk\ Load) + (Fixed\ Expense)}{\{1 - Commisions - Taxes - Profit + [(Investment\ Return) * (1 + \langle 1/PS \rangle)]\}}$$

For the purposes of this paper, we have made some simplifying assumptions that the reinsurer would have the same expense and profit provisions as the insurer, but in reality these values would be different.

Table 8 below lays out the calculation combining the reinsurance cost and the average rate net of reinsurance.

Table 8: Sample Company Average Rate Net of Reinsurance Calculation

Item #	Component	Value	Notes
(1)	Commissions	20%	Selected
(2)	Premium Tax	4%	Selected
(3)	Fixed Expense	$25	Selected
(4)	Trend	2%	Selected
(5)	Trend Length (Years)	2.5	Selected
(6)	Investment Return	2%	Selected
(7)	Premium to Surplus Ratio	2:1	Selected
(8)	Underwriting Profit Provision	5%	Selected
(9)	LAE	10%	Selected
(10)	Average Annual Direct Loss	5,435,490	Derived from Model
(11)	Average Annual Ceded Loss	258,853	Derived from Model
(12)	House Years	19,206	Derived from Sample Portfolio
(13)	Direct Loss Pure Premium	283.01	= (10) / (12)
(14)	Ceded Loss Pure Premium	13.48	= (11) / (12)
(15)	Trended Direct Loss Pure Premium	297.37	= (13) * [1+(4)] ^ (5)
(16)	Trended Ceded Loss Pure Premium	14.16	= (14) * [1 + (4)] ^ (5)
(17)	Trended Direct Loss and LAE Pure Premium	327.11	= (15) * [1 + (8)]
(18)	Trended Ceded Loss and LAE Pure Premium	15.58	= (16) * [1 + (8)]
(19)	Trended Net Loss and LAE Pure Premium	311.53	= (17) – (18)
(20)	Reinsurance Risk Load	37.52	See Table 7
(21)	Net Risk Load	59.48	See Table 7

Filling these values into the reinsurance rate formula laid out above we have the following:

$$Average\ Reinsurance\ Rate = \frac{[(13.48) * (1.02^{2.5}) * (1.1)] + (37.52) + (25)}{\{1 - 0.2 - 0.04 - \ 0.05 + [(0.02) * (1 + \langle 1/2 \rangle)]\}} = 105.54$$

Finally, fitting that into the formula for the average rate net of reinsurance leaves:

$$Average\ Rate\ Net\ of\ Reinsurance = \frac{[311.53] + (59.48) + (105.54) + (25)}{\{\{1 - 0.2 - 0.04 - \ 0.05 + [(0.02) * (1 + \langle 1/2 \rangle)]\}\}} = 677.78$$

6.2.3 Territory Relativities (Traditional Method)

After using the modeled results for the actual insured portfolio to determine the average rate, the process will shift to using a notional exposure portfolio, as described in Section 5.2 above to define territories and their associated relativity factors. The notional portfolio was created by using the geographic locations of the risks in the actual sample company portfolio. Each geographic location was then set to have the same replacement values, building characteristics, and policy conditions. This ensures that any variation in loss costs is solely reflective of differences in the geographic distribution of risk.

This notional portfolio was then run through the same catastrophe model used to generate the overall rate, and the AAL for each location was generated. Each location within the portfolio was then attributed with a number of simple hazard metrics that are commonly reflective of flood hazard. The values for this case included the following:

- Geopolitical boundaries (FSA)
- Distance to the 100-year Flood Hazard Zone, binned within the following ranges:
  - 0 to 0.1 kilometers
  - 0.11 to 0.25 kilometers
  - 0.26 to 0.5 kilometers
  - Greater than 0.5 kilometers
- Elevation, binned within the following ranges:
  - Less than 600 meters
  - 600 to 650 meters
  - 650 to 700 meters
  - 700 to 750 meters
  - 750 to 800 meters
  - 800 to 850 meters
  - Greater than 850 meters

The binned values were judgmentally selected based on the more detailed values within the portfolio. The selected ranges were chosen to eliminate some of the statistical variation within the detailed results.

This process resulted in groupings of locations within the notional portfolio based on largely contiguous territories. Once the locations are grouped appropriately, the appropriate relativity for each territory is determined by comparing the average losses within each territory to the overall average rate, while accounting for credibility.

The credibility criteria used is the simple limited fluctuation credibility standard of 1,082 claims, corresponding to P (probability of random error) =.05 and the 90th percentile of the normal distribution. The number of claims for full credibility is therefore defined as:

$$N = \left(\frac{Z}{P}\right)^2 = \left(\frac{1.645}{0.05}\right)^2 = 1{,}082$$

Since modeled loss estimates are being used, the claims count subject to this credibility criteria are modeled claims estimates. For the purposes of this study, a claim is assumed when losses exceed $500 for a given location. The modeled claims count should account for the uncertainty in the mean damage associated with a given location for each simulated event, and therefore be representative of the expected claims for a given risk over the 10,000 simulated years of catastrophe activity.

Standards for full credibility are calculated in terms of the expected number of claims. It is common to translate these into exposures by dividing by the (approximate) expected claim frequency. Consequently, the number of exposures required for full credibility is equal to 1,082/expected claim frequency.[14]

In the sample company portfolio there are 19,206 locations. Each of these locations is modeled 10,000 times. There are a total of 728,391 claims for the portfolio simulated with the 10,000-year catalog. Therefore, the average number of claims per house year simulated is approximately 38 and the expected claim frequency is calculated as:

$$38/10{,}000 = .0038$$

The number of exposures required for full credibility would therefore be approximately:

$$1{,}082/.0038 = 284{,}737$$

[14] (Mahler & Dean, 2001)

Additionally, if we think of this in terms of confidence intervals around frequency and separately severity, with 10,000 observations or trials at each location the confidence interval width of the frequency estimate is small. Severity is significantly more variable and can only be observed when a loss-causing event impacts an exposure, or after roughly 38 observations.

Where the claims count for a given territory grouping does not exceed the credibility criteria, the average loss within that territory is credibility weighted with the average for the whole geopolitical boundary, across all variations of elevation and distance to the 100-year flood plain. The credibility (Z) is defined as:

$$Credibility\ (Z) = MIN\left(1; \sqrt{\frac{ClaimCount}{1082}}\right)$$

The credibility weighted territory average loss is therefore defined as:

$$\begin{aligned} &Credibility\ Weighted\ Territory\ Average\ Loss \\ &\qquad = Z * (Territory\ Average\ Loss\ Prior\ to\ Credibility\ Weighting) + (1 - Z) * (FSA\ Average\ Loss) \end{aligned}$$

The resultant territory relativities are then compared to the modeled loss estimates for each location in the notional portfolio to determine how well they reflect the expected losses within the groupings. The results of the regression test for this purpose are included in Table 9 through Table 11 below. The F statistic value of 29,283 is sufficiently large to ensure an insignificant within variance compared to between territories. Further, the Multiple R coefficient implies a high degree of correlation between loss cost and territorial assignment.

Table 9: Notional Portfolio Traditional Territory Definition Regression Statistics

Regression Statistics	
Multiple R	.78
R Square	.60
Adjusted R Square	.60
Standard Error	81.29
Observations	19,206

Table 10: Notional Portfolio Traditional Territory Definition ANOVA Result

	df	SS	MS	F	Significance F
Regression	1	193,505,018	193,505,018	29,283	0
Residual	19,204	126,901,605	6,608		
Total	19,205	320,406,623			

Table 11: Notional Portfolio Traditional Territory Definition Coefficients

	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
Intercept	33.62	1	28.34	4.3E-173	31.29	35.94
Territory Definition	0.265	0	171.12	0	.262	.268

The absolute average error of the territories relativities is approximately 20%, and the weighted average coefficient of variation is 9.5%, showing that the territory definitions are appropriate for this use. The resulting territory definitions result in 1,425 unique territories.

In addition to the methodology described here, we have also evaluated a less traditional methodology that leverages non-contiguous territories. This will be discussed in Section 7.5.1 below.

6.2.4 Rating Relativities

An additional notional exposure data set was generated for the purposes of calculating rating relativities for construction type and deductible values.

The notional exposure data set was generated by taking the base risk used for the territory definition analysis, and repeating it nine times at each geographic location, varying either the construction type or deductible value for each additional location record. Three different construction types and five different deductible values were considered in addition to the base characteristics (base characteristics assumed reinforced masonry construction and a $1,000 deductible). All other exposure characteristics were held constant, thus ensuring that any variation in average losses was solely a result of the changes in construction or deductible.

The AAL across all locations was calculated for each rating variation. The respective values were then divided by the AAL for the base risk to determine how much that characteristic would be expected to change the rate for a given policy. The results for that calculation are shown in Table 12 below.

Table 12: Notional Portfolio Calculation of Rating Relativities

Variable	Construction Type	Deductible Value	Base Average Annual Loss	Updated Average Annual Loss	Average Annual Loss Ratio	Standard Deviation of AAL Ratio	CV
Base	Reinforced Masonry	$1,000	210.07	210.07	1.000	0.000	0.000
Construction	Wood	$1,000	210.07	283.31	1.349	0.003	0.002
Construction	Masonry Veneer	$1,000	210.07	254.71	1.212	0.002	0.002
Construction	Unreinforced Masonry	$1,000	210.07	245.16	1.167	0.002	0.002
Deductible	Reinforced Masonry	$2,500	210.07	209.35	0.997	0.001	0.001
Deductible	Reinforced Masonry	1%	210.07	208.65	0.993	0.001	0.001
Deductible	Reinforced Masonry	2.5%	210.07	193.58	0.922	0.012	0.013
Deductible	Reinforced Masonry	5%	210.07	179.20	0.853	0.016	0.019
Deductible	Reinforced Masonry	10%	210.07	157.10	0.748	0.020	0.027

There is very little variation between the ratios shown above, so it was determined that it was unnecessary to evaluate the relativities at a finer geographic resolution than portfolio-wide. This process would likely be required if the rating plan was evaluating a much wider geographic region or was evaluating policies in areas of much greater flood hazard, such as those within the 100-year flood zone.

Through validation of the model being leveraged for this analysis, it was known that there are no explicit dependencies between construction and deductible. This allowed the two characteristics to be evaluated independently for the purposes of this rating plan. The independence was confirmed by testing whether the deductible relativities would be significantly different if they were calculated using a base risk of wood frame rather than reinforced masonry. Wood frame construction was selected due to being the most vulnerable of four construction types considered, while the original base risk of masonry is the least vulnerable. The results of this sensitivity test can be seen in Table 13 below.

Table 13: Notional Portfolio Sensitivity Case Calculation of Rating Relativities

Variable	Construction Type	Deductible Value	Base Average Annual Loss	Updated Average Annual Loss	Average Relativity	Standard Deviation of AAL Ratio	CV
Base	Wood	$1,000	283.31	283.31	1.000	0.000	0.000
Deductible	Wood	$2,500	283.31	282.27	0.996	0.001	0.001
Deductible	Wood	1%	283.31	281.32	0.993	0.001	0.001
Deductible	Wood	2.5%	283.31	262.10	0.925	0.012	0.013
Deductible	Wood	5%	283.31	243.17	0.858	0.016	0.019
Deductible	Wood	10%	283.31	213.80	0.755	0.020	0.027

Comparing the deductible relativities using this sensitivity case to the base case scenario in Table 12 shows that all of the relativity factors are very similar, indicating that there is no inter-dependence between the two variables.

This will not always be the case, however, as many building characteristics are inter-dependent, as are many policy conditions. If inter-dependence within the model was known, or demonstrated through tests such as this, then it would be most actuarially sound to consider all possible combinations of the rating variables being considered, to avoid skewing any of the resulting relativity factors. When many inter-dependent rating variables are being considered at once, such as in the case of a comprehensive mitigation study, the exponential increase in the number of possible combinations can lead to the notional portfolio becoming so large as to make analyzing it with a catastrophe model impractical. In these cases, it is necessary to reduce the number of geographic points being modeled. When making the reduction every care should be taken to ensure the geographic distribution of the rating plan is maintained. This can be accomplished by ensuring that there are representative points from each territory defined for the rating plan.

6.2.5 Off-Balancing

At this stage, the initial base rate has been determined, and then that rate has a full range of adjustments that can be made to account for territory and rating relativity factors. The next step is to adjust the base rate to ensure that the calculated rates for all policies within the portfolio balance to the Initial Average Rate, using the formula below.

$$Final\ Base\ Rate = \ Initial\ Average\ Rate * \left(\frac{Initial\ Average\ Rate}{Average\ Calculated\ Rate}\right)$$

The Average Calculated Rate is the average of the rates for each policy in the portfolio. The rate for each policy is determined by applying all the territory and rating relativity factors as well as an exposure factor to the average rate. In the case of this rating plan, the formula for each policy's rate is as follows:

$$Policy\ Rate = (Average\ Rate) * (Exposure\ Rel.) * (Territory\ Rel.) * (Construction\ Rel.) * (Deductible\ Rel.)$$

The average rate is as calculated net of reinsurance in Section 6.2.2 above, $677.78. The exposure factor is calculated as the replacement value for the building in the policy divided by the base replacement value of $300,000. The territory and rating factor relativities are calculated according to the procedure in Sections 6.2.3 and 6.2.4 respectively.

The formula below shows a sample rate calculation for one policy in the portfolio. The policy is in Territory 1, has a building value of $232,000, is of wood construction, and has a deductible value of $1,000. The policy rate is therefore calculated as follows:

$$Policy\ Rate = (\$677.78) * \left(\frac{232{,}000}{300{,}000}\right) * (0.166) * (1.349) * (1.000) = \$117.38$$

This calculation is repeated for each of the 19,206 policies in the portfolio, and then the average of these rates is taken. The resultant average is $920.63. This then feeds into the off-balancing formula above as follows:

$$Final\ Base\ Rate = \$677.78 * \left(\frac{677.78}{920.63}\right) = \$498.99$$

The resultant base rate is then used as the starting point for calculating the rate for each individual policy. The calculation for the selected policy described above is as follows:

$$Policy\ Rate = (\$498.99) * \left(\frac{232{,}000}{300{,}000}\right) * (0.166) * (1.349) * (1.000) = \$86.41$$

The resulting average across all policies should then balance to the Initial Average Rate of $677.78, which it does in this case.

This demonstrates a relatively simple application of this procedure, as there are only a few relativity factors being applied. In practice, there are likely to be many more factors, but the process should remain unchanged.

6.2.6 Final Rate Pages

At this point, all the calculations to determine the full rating plan are complete. Below is a summary of the rate pages for the plan.

Table 14: Sample Company Rate Pages Base Risk Definition

Variable	Value
Occupancy	Single Family Home
Construction	Reinforced Masonry
Deductible	$1,000
Building Replacement Value and Limit (Coverage A)	$300,000
Appurtenant Structures Value and Limit (Coverage B)	5% of Coverage A
Contents Value and Limit (Coverage C)	70% of Coverage A
Additional Living Expense Limit (Coverage D)	20% of Coverage A
Base Rate	$489.99

Table 15: Sample Company Rate Pages Construction Relativity Factors

Construction	Relativity
Reinforced Masonry	1.000
Wood	1.349
Masonry Veneer	1.213
Unreinforced Masonry	1.167

Table 16: Sample Company Rate Pages Deductible Relativity Factors

Deductible	Relativity
Initial Policy Deductible	1.000
1.0%	0.994
2.5%	0.915
5.0%	0.844
10.0%	0.737

The territory definitions for the rating plan included 1,425 unique territories. Rather than list these all here, they have been included as Appendix C. The lowest-loss territory has a relativity factor of 0.166, while the highest-loss territory has a relativity factor 4.750; 739 of the territories indicate a rate lower than the base rate, while 685 indicate a rate higher than the base rate. One territory shows no difference from the base rate.

6.3 Compare Experience-Based Rates to the Proposed Framework

To demonstrate some of the benefits of leveraging catastrophe models in the context of the sample company, the sample company's actual loss experience (as shown in Table 4 above) was used to generate a similar rating plan. The ratemaking was performed twice, once using the full set of historical experience through 2017, and once using only the experience through 2016, with the 15th year being replaced with the average losses for 2003–2016.

The first step is to calculate the risk load based on these two scenarios of historical loss experience.

Table 17: Sample Company Historical Loss Experience for Rating

	All Years (Scenario 1)		2003–2016 (Scenario 2)	
Year	Claims	Historical Loss	Claims	Historical Loss
2003	7	375,487	7	375,487
2004	103	8,169,837	103	8,169,837
2005	97	8,150,913	97	8,150,913
2006	49	4,103,903	49	4,103,903
2007	29	1,952,638	29	1,952,638
2008	150	13,069,088	150	13,069,088
2009	33	2,588,974	33	2,588,974
2010	10	514,923	10	514,923
2011	74	5,585,134	74	5,585,134
2012	32	1,991,075	32	1,991,075
2013	15	933,819	15	933,819
2014	129	9,634,851	129	9,634,851
2015	2	340,155	2	340,155
2016	76	5,026,550	76	5,026,550
2017	390	33,756,400	*58*	*4,459,811*

Average Annual Losses	6,412,916		4,459,811
Standard Deviation	8,487,818		3,850,158
Reluctance Factor	0.15		0.15
Risk Load	1,273,173		577,524
House Years	19,206		19,206
Pure Premium	333.90		232.21
Risk Load	66.29		30.07

There are no years that breach the reinsurance layers for the sample company, so for the purposes of this exercise we will only consider the direct average rate. Retaining the same reluctance factor and formulas used previously, the direct risk loads and pure premium are calculated in Table 17 above.

We can then fit these values into the same framework as in Section 6.2.1 above.

Table 18: Sample Company Direct Average Rate Calculation Comparison to Historical Experience

Item #	Component	Modeled Loss Results (A)	Scenario 1 (B)	Scenario 2 (C)	Notes
(1)	Commissions	20%	20%	20%	Selected
(2)	Premium Tax	4%	4%	4%	Selected
(3)	Fixed Expense	$25	$25	$25	Selected
(4)	Trend	2%	2%	2%	Selected
(5)	Trend Length (Years)	2.5	2.5	2.5	Selected
(6)	Investment Return	2%	2%	2%	Selected
(7)	Premium to Surplus Ratio	2:1	2:1	2:1	Selected
(8)	Underwriting Profit Provision	5%	5%	5%	Selected
(9)	LAE	10%	10%	10%	Selected
(10)	Average Annual Direct Loss	5,435,490	6,412,916	4,459,811	(A) = Derived from Model; (B) & (C) = See Table 16
(11)	House Years	19,206	19,206	19,206	Derived from Sample Portfolio
(12)	Direct Loss Pure Premium	283.01	333.90	232.21	= (10) / (11)
(13)	Trended Direct Loss Pure Premium	297.37	350.85	243.99	= (12) * [1 + (4)] ^ (5)
(14)	Direct Loss and LAE Pure Premium	327.11	385.93	268.39	= (13) * [1 + (9)]
(15)	Risk Load	70.50	66.29	30.07	(A) = See Table 5 (B) & (C) = See Table 16
(16)	**Direct Indicated Average Rate**	**571.10**	**644.90**	**437.11**	**See Formula from Section 6.2.1**

The large fluctuations in losses from Scenario 1 to Scenario 2 demonstrate the challenge of rating catastrophe perils using historical loss experience. A period of 14 years from 2003–2016 yields a rate that is 23% lower than the long-term average rate indicated by the catastrophe model, and adding a single additional year to that experience period increases the indicated average rate by 48%. This rate instability is challenging for both insurers and policy holders, and could result in significant shocks to policyholder's surplus after years of inadequate rates.

The losses in year 2017 represent slightly below a 50-year return period. This may seem like an extreme example to occur within a 15-year experience period, but there is approximately a 26% chance of experiencing a 50-year return-period loss in any 15-year period.

It would of course be a remarkable coincidence that the one year of especially large loss happens to be the most recent in the 15-year experience period. It simply illustrates that, when dealing with a catastrophe peril, many years can be experienced with no observed losses from the "tail" of the loss distribution. It should be realized that, in practice, it does not matter where that most extreme result occurs. The difficulty is the same in all cases. Not having to interpret the historical experience, deciding whether the highest observed loss should be discounted as abnormally high or if it just gives a glimpse of even larger losses that should be expected, is one of the benefits of using the catastrophe model.

In addition to ensuring rate stability by leveraging a catastrophe model to determine long-term average losses for the portfolio, the model provides the added benefit of simulating the impact of rating variables and geographic variation in loss estimates without bias. Utilizing the historical experience for this purpose would result in significant bias and sampling errors in the calculation of the relativity factors. While we did not have detailed historical claims experience for the sample company with which to generate historical-based relativities, we did leverage modeled results for the sample company's actual portfolio to illustrate the issues that bias can introduce when evaluating relativities. By comparing the average losses per exposure for different categories of policies within the portfolio, we generated the results seen in Table 19.

Table 19: Sample Company Rating Relativities Calculation Based on Actual Portfolio

Variable	Construction Type	Deductible Value	Average Limit	Locations	Average Annual Loss	Average Loss per Exposure	Average Relativity
Base	Reinforced Masonry	$750	335,806	3,261	288	267	1.00
Construction	Wood	$750	336,249	2,059	288	266	1.00
Construction	Masonry Veneer	$750	334,778	2,518	287	267	1.00
Construction	Unreinforced Masonry	$750	336,178	1,623	285	262	0.99
Deductible	Reinforced Masonry	$500	280,619	1,710	288	317	1.00
Deductible	Reinforced Masonry	$1,000	447,885	1,633	298	214	1.03

There are a number of unintuitive results resulting from the bias in the portfolio. You see virtually no variation in average losses between different construction codes, and there are actually inverted relativities for an increased deductible value. While some of this bias can be removed through standard actuarial techniques, leveraging a catastrophe model with notional exposure makes it possible to eliminate the bias much more simply.

Finally, the model generates many thousands of simulations of annual catastrophe activity. There is no way to generate a historical experience set that would represent such a robust set of losses. This limitation leads to credibility concerns when certain areas being rated have little or no historical experience with which to generate a rate.

Section 7 Additional Areas for Exploration

7.1 Underwriting Decisions

This paper addresses methods that can be used to determine the technical or actuarial price. As is the case with the actual or market pricing of catastrophe risk, insurers will need to consider prices and/or coverages being offered by competitors in various markets. The technical price in aggregate and by territory or other rating variables is highly dependent on accuracy and appropriateness of the underlying catastrophe model. Other competitors will often be using different catastrophe models. When significant differences in market prices appear between respected competitors the actuary should assess if this points to a marked divergence in how competing models may be assessing the impact of rating variables and use this information as part of the actuarial control cycle.

Even if the catastrophe model is not explicitly incorporated into an insurer's pricing algorithms, the results of catastrophe models can be used to refine underwriting guidelines to prohibit offering specific coverages in high-risk territories or on property with or without certain mitigating attributes.

7.2 Portfolio Management

Capital is needed to underwrite a portfolio of policies, and capital comes at a cost. The method described in this paper allows for that cost by including a risk load, the cost of reinsurance, and a profit provision. Over time those costs can change as the portfolio of policies changes. When there are large changes to a portfolio the resulting cost changes merit consideration.

In considering the addition of a risk or set of risks an insurer can be mindful of how the addition will affect their overall risk profile. Changes may be gradual, as is usually the case when writing individual homeowners' policies. Change can be more pronounced if larger risks are being written or a sizeable book of business is acquired and added to the portfolio. Insurers may wish to evaluate the marginal risk caused by the portfolio change. The marginal risk is the amount that a risk metric changes because of the addition or deletion of insured exposures.

A catastrophe model typically can provide the marginal risk metric by aligning the estimated event losses for the marginal risk or set of risks with the corresponding event losses for the existing portfolio.

An insurer may find that the addition of a risk or a set of risks pushes up their risk profile to where a rating agency would have concern or beyond a level that is consistent with the design of their reinsurance protection. The insurer may decline the risk, modify the coverage terms, or possibly lay off some of the risk through additional reinsurance, e.g. a facultative placement. It may also be that the added exposures have minimal correlation with the existing portfolio or are located in areas with less concentration of risk; in which case the marginal risk will not be a concern. An approach in which notional exposures, similar to what were used in the ratemaking example, is added to the existing portfolio for modeling purposes can also be useful for evaluating growth and marketing plans.

The ratemaking method described in Section 5 above provides for a risk load based on the set of modeled policies. As a rating plan, it does not dynamically account for changes in the risk of the portfolio from writing policies that may be growing the portfolio. If one wishes to know how much risk load to incorporate for a larger or smaller portfolio, you can remodel the portfolio. The modeling process itself will reflect the correlation (or lack of correlation) of the anticipated change in portfolio, and the difference in risk load on the changed portfolio can be seen by comparing to the original risk load.

Any geographic concentrations of policies can deserve attention because they will drive the overall level of risk and will also affect the reinsurance cost in the long run. Insurers may wish to control where growth is occurring, by either market price adjustments or by underwriting policy, to manage overall portfolio risk. Insurers also can get more refined in how risk loads and reinsurance costs are built into the rates, as described below in Section 7.6.

7.3 Variations to Proposed Methodology for Different Ratemaking Situations

The framework described above is applicable to an insurer pricing a new endorsement for an existing portfolio of homeowners' policies. The principles described therein are broadly applicable to all ratemaking situations, but there are some differences.

7.3.1 Rating a Standalone Policy

The example presented assumed an endorsement was being rated. To price a standalone policy, the differences would be relatively minor, but would include differences in the primary rating relativities and the base deductible definition. You would also have more flexibility in underwriting criteria, which could vary widely for a standalone policy.

7.3.2 Ratemaking for an Entirely New Program

The example presented assumed an existing portfolio as the basis of the new policy offering. If the creation of a rating plan for a new program was to be considered, it would be necessary to create a hypothetical current portfolio. This could be done by specifying underwriting criteria and desired rating parameters, and then randomly simulating a desired portfolio. Consideration should be given to the expected geographic distribution, and the expected percentages of different rating parameters being included in the portfolio.

7.4 Variations to the Proposed Methodology for Different Perils

The examples described above were focused on ratemaking for the flood peril. While the framework would apply generally across all catastrophe perils, there are some variations that should be considered based on the nature of the peril being modeled.

7.4.1 Severe Thunderstorm

The flood peril has a lot of variation in risk from one location to the next, due to the significant impact of small changes in things such as elevation and slope within small areas. Thus, floods frequently impact very specific areas. Severe thunderstorms (consisting of tornados, hail, and straight-line winds) similarly impact very specific areas when they occur, but that is not directly attributable to changes in hazard within a relatively small area, but rather is simply the random chance of where these small isolated cells of storm activity pass through.

Because of the increased randomness of severe thunderstorm activity, combined with the limited size of impacted areas, it is most appropriate to take a less granular approach to developing territory definitions. Many geographic locations should be modeled and then aggregated in contiguous areas to ensure appropriate smoothness of the resultant loss costs.

Since the primary damaging characteristics of a severe thunderstorm are wind (both cyclonic and straight-line) and hail, roof characteristics have a much bigger impact on losses than they do for flood, where elevation and basement-related characteristics have more impact.

7.4.2 Winter Storm

Winter storms generally impact a fairly large area, and the geographic variations in risk are also at a fairly large scale, with the exception of extreme elevations where increased snow load and wind expectations are both present. Because of this, a coarser approach to aggregating territories would be appropriate.

The primary damaging characteristics of winter storms are wind, winter precipitation (snow, ice pellets, or freezing rain) and freezing temperatures. Thus, roof characteristics have a much bigger impact on losses than they do for flood, as well as insulation-related features.

7.4.3 Hurricane

Hurricanes generally impact a fairly large area, but the geographic variations in risk are very highly correlated with distance to coast, as the events typically undergo significant weakening as they move over land. Because of this, it is appropriate to evaluate a higher degree of regional variation in territories along the coast, and a coarser resolution inland.

The primary damaging characteristics of hurricanes are wind, storm surge, and precipitation-induced flooding. For the wind peril, roof characteristics have a much bigger impact on losses than they do for flood.

In contrast to winter storm and severe thunderstorm, hurricane winds can have a much longer duration of highly damaging winds, which enhances the importance of opening protections and missile sources in the area.

The surge and precipitation components for hurricanes are very similar to the inland flood sources considerations described herein.

7.4.4 Earthquake

Earthquakes generally impact a fairly large area, and while the risk is concentrated in areas of known faults, there is potential for loss everywhere.

Further, significant increases in risk can occur due to highly localized soil conditions, where some soils have the potential to liquefy due to ground motion and cause extensive damage, even far away from the source earthquake. Thus, territory definitions should take into consideration local soil conditions in addition to proximity to known faults and seismic regions.

Earthquakes can cause damage from a range of sub-perils including ground shaking, liquefaction, landslide, fire-following, sprinkler leakage, and tsunami. The ground motion perils are heavily dependent on foundation type, seismic response features, and the number of stories.

Fire-following is highly dependent on the building density, and a tsunami responds similarly to the flood peril.

7.4.5 Wildfire

The majority of wildfires are either contained to a relatively small area, or burn large areas of undeveloped forest, so the majority of damage is concentrated in the Wildland–Urban Interface (WUI), which is the area where urban development meets the forest. Territories should take surrounding fuels into consideration, and similar to severe thunderstorm, should include enough sampling variation to ensure the random occurrence of urban-area fires is aggregated across a wider area.

Fire damage is heavily driven by the flammability of the structure. Due to the potential for rapid spread of fires once they encroach on concentrated areas of buildings (such as suburban developments), community-based mitigation techniques have significant impact.

7.5 Potential Enhancements to the Proposed Methodology

7.5.1 Non-Contiguous Territory Relativities

While the territories defined in Section 6.2.3 above are statistically fit for purpose, there are limitations in the methodology of grouping territories this way. It results in a large number of territories, and it is possible to refine the accuracy of the territory definitions. We do this by eliminating the need for contiguity in the territory definitions.

In this approach, the same notional portfolio results are utilized, but rather than evaluating groupings at the FSA level, we instead start by grouping results down to the postal code level, retaining the same elevation and distance to flood zone ranges. Alternatively, any defined grid of desirable specificity could be constructed, which may be preferable for a non-metropolitan region. This initially results in significantly more groupings (increasing by a factor of over 5 in this case). The additional refined groupings are rank ordered from highest to lowest average losses. Finally, an iterative process is followed to assign the highest average loss grouping to Territory 1, and evaluate the coefficient of variation between sub-groupings within the territory and the credibility criteria of the territory. That process is iterated, adding a new sub-grouping to the territory and re-calculating the credibility and coefficient of variation, until either full credibility is reached or the coefficient of variation exceeds 1%. Once those thresholds are met, the next-highest loss sub-grouping becomes Territory 2, and the process is repeated until all sub-groupings are accounted for and thus territories are fully defined.

Applying this methodology to the same notional exposure results that were analyzed in Section 6.2.3 resulted in only 62 territories (as opposed to 1,425) and reduced the absolute average error from 20% to 8.5%. Regression statistics based on this sample can be found below. The multiplicative inverse of the F statistic, like that corresponding to the traditional method, rounds to zero. The correlation coefficient is higher at approximately 90%.

Table 20: Notional Portfolio Traditional Territory Definition Regression Statistics

Regression Statistics	
Multiple R	.89
R Square	.79
Adjusted R Square	.79
Standard Error	59.50
Observations	19,206

Table 21: Notional Portfolio Traditional Territory Definition ANOVA Result

	df	SS	MS	F	Significance F
Regression	1	252,418,135	252,418,135	71,298	0
Residual	19204	67,988,487	3540		
Total	19205	320,406,623			

Table 22: Notional Portfolio Traditional Territory Definition Coefficients

	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
Intercept	483.94	1	435.24	0	481.76	486.12
Territory Definition	-9.13	0	-267	0	-9.20	-9.06

For the flood peril, this more refined approach can have substantial benefits. As has been mentioned previously in this paper, flood losses can vary significantly within a small geographical area, making the non-contiguous approach likely more appropriate.

This approach could be improved with the addition of notional data at points other than those in the current portfolio facilitating the writing of new business and accuracy of the estimates. This consideration is addressed in Section 7.5.2 below.

Due the fact that the non-contiguous method may not be applicable to catastrophe perils that are less location-specific than flood, the proof of concept utilized the traditional method. However, the process described in Sections 6.2.4 through 6.2.6 would remain unchanged in concept, regardless of the specifics of the territory definition process.

7.5.2 Use of Fully Notional Data

The rating plan for the sample company leveraged a notional portfolio that had geographic locations based on the locations in the existing portfolio. This has the potential to lead to gaps in the territory definitions for areas that are not currently represented in the portfolio.

Using a full set of notional data that included areas not included in the current portfolio would enable the determination of more accurate territory configurations. This would be true for both contiguous and non-contiguous methods described in this paper. Notional data could be generated for all the geographical areas the insurer wished to write in, by using all possible FSAs or postal code areas, or by leveraging a gridded set of coordinates. With a complete set of notional data, credibility would be improved and contiguous territories could be established using a more refined definition such as subarea and elevation and distance ranges.

Additional notional data runs may improve the accuracy of primary relativities as well, although the extent of improvement is likely to be limited given the minor variation using the portfolio locations.

7.5.3 Evaluation of All Combinations of Possible Rating Variables

As described in Section 6.2.4 above, for the variables in consideration for the sample company's rating plan, and the model being leveraged to generate the losses, there were no inter-dependencies that needed to be considered. This will not always be the case, as there are many building and policy characteristics that are going to have inter-dependencies. To avoid double-counting the impact of related characteristics, it would be more robust to consider all possible combinations of rating variables being considered. This has the potential to result in significantly larger notional exposure and loss data sets, but that can be managed through the reduction in the number of geographic points being modeled in the notional portfolio.

7.5.4 Setting Rates/Reinsurance Allocation for Higher-Risk Properties

The sample company included for this proof of concept eliminated the highest-risk properties from consideration for the portfolio. This was determined for affordability concerns related to the fact that the hypothetical rating plan is intended to be offered as an endorsement to a homeowner's policy. Therefore, the average premium was restrained to be a fraction of the average homeowner's premium. This will not always be possible or desired, and there are some considerations that should be taken into account when rating such policies.

It is common for insurance coverage for higher-risk properties to incorporate or even require a number of adjustments to mitigate that risk for the insurer. For example, flood deductibles for high-risk properties may be offered at larger amounts or percentages than the standard homeowners' deductibles, and coverage limits for flood are frequently lower than other perils. The policy structure and rating plan should therefore be customizable enough to ensure affordability of insurance for high-risk properties without jeopardizing actuarially sound rates.

It is also important to be very careful with modeled data that generates very high loss costs for particular regions or policies, to ensure that the modeled result is valid. For example, for the flood peril, a grid cell, postal code centroid, or street address geocode may inadvertently be placed within a water body. This would greatly inflate the loss estimates generated by the model, so care should be taken to ensure accurate geocoding of those risks. It may be worth operating outside the context of a structured rating plan to have risk-specific rating for the highest-risk policies, so that detailed consideration can be given to the geographic location of the property, and any loss mitigation efforts that have been employed.

7.5.5 Encouraging Risk Mitigation

The only building characteristic considered for the sample company was construction relativity. There are many more building features that can serve to mitigate losses from catastrophe perils such as flood. Providing premium credits for mitigation efforts can help to encourage their adoption, and can help reduce the overall risk to the insurance company.

For the flood peril in particular, mitigation efforts can include property-specific mitigation such as first-floor elevation and service equipment protection, as well as community-based mitigation efforts such as levees systems. Provided the catastrophe model being utilized can capture these features and their relative impact appropriately, they can be rated in very similar ways to the construction relativities for the sample company. Care should be taken to determine the independence (or lack thereof) of any mitigation features in relation to other building or policy characteristics within the rating plan, and adjustments made to model all possible combinations of those variables if they are indeed inter-dependent.

7.5.6 Allocation of Risk Load and Reinsurance Cost

In a more geographically dispersed portfolio (for example, province- or country-wide), or where higher-risk areas are included in the portfolio, the risk load and reinsurance cost may be driven by select areas of accumulated exposure or high levels of risk. In these cases, it would be advisable to allocate the risk load and the cost of reinsurance back to those areas appropriately. When performing this process, risk loads for each territory should aggregate to the overall portfolio risk load. The aggregate risk load for all territories should equal the portfolio risk load. This will often not be the case due to correlation considerations.[15] This can be accomplished by applying a scaling factor to the risk load of each territory which would be proportional to the correlation coefficient. Rates will be balanced but without adjustment the risk load may be over-emphasized in the rates.

7.6 Variations in Risk Load Pricing

The method used to estimate the risk loads in our hypothetical example for both the primary and reinsurer rates was the traditional method widely used in determining catastrophe risk loads:

$$Risk\ Load = Expected\ Loss + (Risk\ Load\ Factor) * \sigma$$

The Kreps paper was the original basis for this technique. In practice the technical determination of the risk adjustment factor involves judgment of risk perception and tolerance (the confidence interval) even prior to being a consideration in the negotiations between the broker, reinsurer, and primary insurer in procuring coverage.

Property catastrophe model output provides a good technical starting point in determining catastrophe risk loads. In addition to the standard deviation, model output includes other variables that may be used in the risk load calculation. A brief overview of a few of these variables, the relative advantages and disadvantages of each, and an example calculation based on the sample company's modeled loss estimates are listed below.

Standard deviation

- Advantages
  - Ease of calculation, included in model output
  - Practitioners have experience in pricing with this method, pricing history to draw upon
  - Pure risk measure
- Disadvantages
  - Inadequate emphasis on events in the tail of the distribution (transformation of the distribution or use of a distortion measure may mitigate this concern but will add significant complication)
  - Does not satisfy all the coherence conditions
  - Includes positive and negative outcomes (the use of a conditional standard deviation confined to outcomes greater than the mean would address this concern)

[15] (Burke, 2009)

The standard deviation is calculated as follows:

$$Standard\ Deviation = \sqrt{\frac{\sum(X-\mu)^2}{n-1}}$$

Where X = each simulated year's total estimated losses, μ = the modeled AAL, and n = the number of simulated years in the catastrophe model catalog. There are 10,000 simulated years in the catalog being used by the sample company, and the AAL is $5,435,547 on a direct basis.

VAR ϴ = least favorable outcome from the (1-ϴ) percentile of worst outcomes

- Advantages
  - Widely used metric in banking and finance
  - Ease of determination, included in model output
  - Can explicitly represent probability of ruin or default, which is a significant solvency consideration to insurers
- Disadvantages
  - Focuses on a single point distribution resulting in inadequate emphasis on the tail
  - Does not satisfy the principals of coherence
  - Ignores the rest of the distribution, non-tail

VAR ϴ is equivalent to the exceedance probability of ϴ%. In this case, we are interested in the total losses in each year, so the AEP Curve would be utilized. The AEP Curve would be generated by taking the sum of event losses in each simulated year, ranking the years in descending order, and then calculating the Aggregate Exceedance Probability as the rank divided by the total number of simulated years. Table 23 below demonstrates the derivation of the EP Curve based on the sample company's modeled loss experience.

Table 23: Sample Company Aggregate Exceedance Probability Curve Derivation

Year ID	Direct Modeled Loss ($)	Rank	Exceedance Probability	Return Period
3782	139,643,617	1	0.01%	10,000
3513	129,084,766	2	0.02%	5,000
1043	96,579,686	3	0.03%	3,333
4886	95,703,031	4	0.04%	2,500
5768	88,055,675	5	0.05%	2,000
2712	87,732,403	6	0.06%	1,667
8976	85,628,571	7	0.07%	1,429
2209	83,131,713	8	0.08%	1,250
….	….	….	….	….
9453	45,448,618	100	1.00%	100
….	….	….	….	….
9532	0	10,000	100.00%	0

Based on Table 23 above, the VAR 1% would be equal to $45,448,618.

TVAR ϴ = average of all outcomes in the (1-ϴ) percentile of worst outcomes

- Advantages
  - Ease of determination, included in model output
  - Fully reflects all losses in the tail of the distribution

    - A probability of default- or ruin-related solvency measure
    - Coherent risk measure

- Disadvantages
    - Ignores all distributional risk not specific to the tail
    - Includes the risk of losses which will likely not be paid by the insurer because of potential insolvency
    - Coherence may not always be a significant concern[16]
    - Less frequently used measure than the standard deviation and VAR

The TVAR ϴ is calculated based off of the EP Curve, by averaging the total losses in all years at or above an exceedance probability of ϴ%.

Table 24: Sample Company Aggregate TVAR Derivation

Year ID	Direct Modeled Loss ($)	Rank	Exceedance Probability	Return Period	TVAR ($)
3782	139,643,617	1	0.01%	10,000	139,643,617
3513	129,084,766	2	0.02%	5,000	134,364,192
1043	96,579,686	3	0.03%	3,333	121,769,356
4886	95,703,031	4	0.04%	2,500	115,252,775
5768	88,055,675	5	0.05%	2,000	109,813,355
2712	87,732,403	6	0.06%	1,667	106,133,196
8976	85,628,571	7	0.07%	1,429	103,203,964
2209	83,131,713	8	0.08%	1,250	100,694,933
….	….	….	….	….	….
9453	45,448,618	100	1.00%	100	60,130,295
….	….	….	….	….	….
9532	0	10,000	100.00%	0	5,435,490

Based on Table 24 above, the TVAR 1% would be $60,130,295. You can also note that the TVAR 100% is equivalent to the AAL for the portfolio of $5,435,490.

Aside from the more traditional risk loading measures described above, most other methods are theoretical and often based on prevailing financial valuation theories. For instance, actuaries have proposed various financial methods, including the arbitrage pricing and options theories, among other methods. These methods have been historically used in applications pricing insurance risk for non-catastrophic coverages in long-tailed lines such as workers' compensation and automobile liability using a discounted cash flow analysis. These methods are more complicated than those reviewed above and practitioners have had difficulty employing these methods in practice due the extent of non-systematic risk and lack of history of the returns investors should expect for this type of risk. Perhaps the growing catastrophe bond market will allow for further exploration of these topics.

[16] (Venter, 2010)

Section 8 Conclusion

Through our research we have discovered that actuaries across several Canadian insurers surveyed are using a wide range of methods in their ratemaking and underwriting practices. Some are using the results of catastrophe models and some are not. Even within any insurer, various methods are being used across several types of perils.

This paper has attempted to start to bridge the knowledge gap that exists for actuaries who are incorporating, or are beginning to incorporate, results from catastrophe models into their pricing and underwriting strategies for Canadian insurers. We have done this through outlining a comprehensive approach to ratemaking for a portfolio of flood insurance risks using catastrophe model output. To support the proposed ratemaking approach, we have provided a detailed proof of concept by deriving pricing for a sample company portfolio of risks using the results of an actual catastrophe model.

Most importantly, we have illustrated that the use of catastrophe models can result in a more robust ratemaking and underwriting approach than the use of historical loss experience alone. The benefits of incorporating catastrophe models include:

- Addressing the problems with utilizing historical claims experience for perils with less frequent events and potential for high severity that is unlikely to yield a stable rate reflective of the long-term loss expectation. Historical claims data can be sparse and outdated and lose its relevance over time. Peril patterns can also change over time, along with the change in the placement and concentration of risks for an insurer.
- Providing a range of potential catastrophe loss scenarios, representative of their probability of occurrence. This provides a long-term view of the catastrophe risk that can be averaged to set more stable pricing.
- Ability to eliminate any exposure bias in the calculation of relativity factors using a hypothetical set of policies known as notional exposure data. Catastrophe models present an enhanced ability to develop rating plans that incorporate several factors and are more robust in creating rate differentials.
- Removing the constraints of evaluating loss based only on actual exposures, such as an in-force book of business. Nominal exposures can be the subject of modeling, thereby providing a view of anticipated losses for a set of potential exposures that may arise in future for a company.
- Providing for increased accuracy and stability of risk load and reinsurance metrics.

The proposed approach and example outlined in this paper provides a detailed and robust analysis of ratemaking that an insurer may incorporate. We have also provided commentary on other considerations that pricing actuaries need to make as they look to enhance or implement approaches that are similar. Specific circumstances of each company will differ.

By providing a large set of simulated events, catastrophe models give an effective means of exploring the tail probabilities, the frequency and severity of the events that are least likely to have been observed in history. Thousands of events that can reasonably be anticipated but have not been observed can be included in the event catalog. Sensitivity testing is one of the benefits of working closely with a catastrophe model.

Overall, the use of catastrophe modeling provides a more complete representation of the risk for catastrophe perils, which can help companies in the development of rates and the management of risk and its many dimensions.

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Appendix A — Survey Questions for Canadian Insurers

The following is the question script used for a survey of current Canadian insurer catastrophe ratemaking practices:

1. Please describe your current catastrophe ratemaking practices.
    a. What perils are considered? Are different methods used for different perils?
    b. Are you using historical experience data? If so, how many years of experience? If so, how is historic data adjusted to current levels?
    c. Are you using a catastrophe simulation model in your ratemaking exercises? If not, have you contemplated using a third-party simulation model, and why?
    d. How are new perils, regions, or LOBs dealt with when there is a lack of experience/exposure data?
    e. For how long have you been using the current approach?
    f. Do you foresee changes to the current approach?
    g. What problems or issues are posed by your current practices?
    h. Do you incorporate an explicit provision for the cost of capital held for catastrophic events? If so, how?
2. How do you manage and monitor the catastrophe risk in your insurance portfolio? What risk transfer mechanisms do you use? How do you manage concentration risk?
3. How do you incorporate any additional capital requirements for catastrophe risk into your ratemaking exercise?
4. How is catastrophe risk incorporated into your underwriting decisions and processes?
5. If the research paper were to incorporate an example of a catastrophe risk modeling approach with industry or corporate experience is there a specific peril and/or region of the country you would most like to see?
6. Some catastrophe models do not have a Canada-specific component at this time. Would the research paper be of more value to you if an example used actual Canadian catastrophe model output for a specific peril (e.g. wind) or if it used US catastrophe model output as a surrogate for a specific peril (flood)?
7. What would you like to be doing to incorporate catastrophe models in your rating that is currently not being done? If tools could be made readily available, what would you be looking for to enhance your capabilities in this area?

Appendix B — Summary of Literature Review

The following is a list of the literature reviewed (author indicated in parenthesis) as part of the research portion of the project along with a brief synopsis of each document.

1. Standards of Practice (Canadian Actuarial Standards Board)

   This paper includes the standards of practice for actuaries working within the P&C insurance field in Canada. Standards of practice related to reserving, financial reporting, and pricing/ratemaking are included. Section 2600 addresses P&C ratemaking.

2. Actuarial Standards of Practice (US Actuarial Standards Board)

   The review included the following Actuarial Standards of Practice (ASOPs) applicable for US actuaries with specific focus paid to sections related to P&C ratemaking:

   a. ASOP 1 Introductory Standard of Practice
   b. ASOP 12 Risk Classification in P&C Ratemaking
   c. ASOP 29 Expense Provisions in P&C Ratemaking
   d. ASOP 38 Using Models Outside the Actuary's Area of Expertise
   e. ASOP 39 Treatment of Catastrophe Losses in Property/Casualty Insurance Ratemaking

3. Statement of Principles Regarding Property and Casualty Insurance Ratemaking (Casualty Actuarial Society)

   This paper identifies and describes the principles applicable to the determination and review of property/casualty insurance rates. It provides definitions of terms and goes on to discuss considerations related to rating principles. It is short in length (four pages) and provides basic reading for anyone working with ratemaking.

4. A Modern Architecture for Residential Property Insurance Ratemaking (John W. Rollins)

   Abstract: This paper argues that obsolete rating architecture is a cause of decades of documented poor financial performance of residential property insurance products. Improving rating efficiency and equity through modernizing of rating and statistical plans is critical to the continued viability of these products. The author explains an architecture and techniques for ratemaking homeowner products in a hurricane-prone state.

5. Catastrophe Pricing: Making Sense of the Alternatives (Ira Robbin)

   Abstract: This paper examines different ways of pricing catastrophe coverage for reinsurance treaties and large insurance accounts. While all the methods use catastrophe loss simulation model statistics, they use different statistics and different algorithms to arrive at indicated prices. This paper provides the reader with the conceptual foundations and practical insights for understanding the alternative approaches.

6. Catastrophe Ratemaking Revisited: Use of Computer Models to Estimate Loss Costs (Michael A. Walters & François Morin)

   Abstract: Recent developments in computer technology have significantly altered the way the insurance business functions. Easy access to large quantities of data has rendered some traditional ratemaking limitations obsolete. The emergence of catastrophe simulation using computer modeling has helped actuaries develop new methods for measuring catastrophe risk and providing for it in insurance rates. This paper addresses issues associated with these methods and provides actuaries, underwriters, and regulators with an understanding of the features and benefits of computer modeling for catastrophe ratemaking.

7. Financial Pricing Models for Property-Casualty Insurance Products: The Target Return on Capital (Sholom Feldblum & Neeza Thandi)

   This paper provides details on how financial pricing models can be used to price property and casualty insurance risks. It works through several examples to illustrate how the approach would work.

8. Incorporating a Hurricane Model into Property Ratemaking (George Burger, Beth E. Fitzgerald, Jonathan White, & Patrick B. Woods)

   Abstract: This paper explains the procedures used to incorporate a hurricane model into the development of state loss costs for personal and commercial properties. It explains why a modeling approach should be used to estimate losses for hurricane perils and discusses the procedures and items included in the ratemaking process.

9. Reinsurer Risk Loads from Marginal Surplus Requirements (Rodney Kreps)

   This paper provides the reader with an analysis of the use of the return on the marginal surplus committed to support the variability of a proposed reinsurance contract.

10. The Competitive Market Equilibrium Risk Load Formula for Catastrophe Ratemaking (Glenn G. Meyers)

    Abstract: The catastrophic losses caused by Hurricane Andrew and the Northridge Earthquake are leading many actuaries to reconsider their pricing formulas for insurance with a catastrophe exposure. Many of these formulas incorporate the results of computer simulation models for catastrophes. In a related development, many insurers are using a geographic information system to monitor their concentration of business in areas prone to catastrophic losses. While insurers would like to diversify their exposure, the insurance-buying public is not geographically diversified. As a result, insurers must take on greater risk if they are to meet the demand for insurance. This paper develops a risk load formula that uses a computer simulation model for catastrophes and considers geographic concentration as the main source of risk.

11. Basic Ratemaking (Geoff Werner & Claudine Modlin)

    This paper is core reading for the Casualty Actuarial Society exam syllabus. It describes techniques used by actuaries in the ratemaking process. Chapter 6 focuses on losses used in the ratemaking process and includes a section relating to catastrophes.

12. Atmospheric Perils Megadisaster Year in Canada – Are You Prepared? (AIR Worldwide)

    This article provides an overview and analysis of simulated losses for catastrophic events from three atmospheric perils in Canada. The three perils reviewed are tropical cyclone, severe thunderstorm, and winter storm.

13. Uncertainty in Estimating Commercial Losses – and Best Practices for Reducing It (Dr. Vineet Jain)

    This eight-page article “discusses the complexities inherent in commercial properties and how these complexities translate to uncertainty in modeled losses.” It recommends a list of best practices for estimating commercial losses to reduce the uncertainty in modeling commercial loss.

14. Blending Severe Thunderstorm Model Results with Loss Experience Data – A Balanced Approach to Ratemaking (David A. Lalonde)

    The article “puts the 2011 season in context of past and potential losses and discusses how the AIR Severe Thunderstorm Model for the United States can be used in ratemaking to manage the high volatility in insured losses.

    “AIR’s Severe Thunderstorm Model provides a reliable and stable view of severe risk and avoids shifts in loss costs caused by the volatility in loss experience data. But company claims data is valuable and should not be discounted entirely either. AIR's ratemaking approach integrates both, which properly accounts for tail risk and prevents volatility at granular levels of geography."

15. An Analysis of the Underwriting Risk for DFA Insurance Company (Glenn G. Meyers)

    This paper provides analysis of how to assess the financial condition of an insurance company, based on a fictional insurance company example set up by the Casualty Actuarial Society in 2001. This paper is only tangentially related to catastrophe modeling as it is one area that is required for the Dynamic Financial Analysis structure that the author describes. It has less applicability to the research and topic of our paper than other readings.

16. Report of the Catastrophe Modeling Working Party (GIRO Vienna September 2006)

    This sixty-page paper provides a description of catastrophe models starting with their construction and a timeline of when the major catastrophe model vendors emerged in the market. Differences in models are discussed as well as model data and limitations. Overall the aim of this paper is to describe the varying use of catastrophe models in the insurance industry and impress upon the reader that models should be well understood by users.

17. Catastrophe Exposures & Insurance Industry Catastrophe Management Practices (American Academy of Actuaries Presentation)

    This paper was produced by the American Academy of Actuaries in response to a request by the National Association of Insurance Commissioners for technical assistance regarding how insurance companies' management of catastrophe risks and risk transfer mechanisms.

18. Rethinking Catastrophe Risk Management at the Point of Underwriting (Michael Gannon)

    This short (three-page) article highlights the need for more comprehensive catastrophe risk management and emphasizes that comprehensive catastrophe risk management entails managing at the point of underwriting as well as overall portfolio risk management.

19. The Role of the Underwriter in Insurance (Lionel Macedo)

    This paper provides a general outline of an underwriter's role in the insurance industry and briefly describes each task that an underwriter typically performs. The main role of an underwriter is to create a large pool of homogeneous risks for an insurance company, which is done by assessing acceptance conditions, detecting applicant misinformation, and classifying risks.

20. Catastrophe Risk Management at the Point of Underwriting (Scott Amussen & Bill Churney)

    This article argues the case for increased use of modeling and improved sophistication of that modeling at the point of underwriting. The paper concludes that probabilistic catastrophe analysis has progressed considerably to a point that it can be incorporated into a fully automated underwriting environment, enabling the underwriter to make better risk selection decisions.

Appendix C — Sample Company Territory Relativity Factors (Traditional)

Terr.	Rel. Factor
1	0.166
2	0.189
3	0.199
4	0.202
5	0.220
6	0.221
7	0.225
8	0.226
9	0.236
10	0.236
11	0.237
12	0.239
13	0.240
14	0.243
15	0.244
16	0.246
17	0.246
18	0.253
19	0.256
20	0.263
21	0.265
22	0.269
23	0.273
24	0.275
25	0.280
26	0.282
27	0.283
28	0.284
29	0.284
30	0.284
31	0.288
32	0.288
33	0.298
34	0.300
35	0.303
36	0.306
37	0.308
38	0.308
39	0.309
40	0.310
41	0.310
42	0.310
43	0.311
44	0.312
45	0.313
46	0.314
47	0.314
48	0.315
49	0.315
50	0.318

Terr.	Rel. Factor
205	0.512
206	0.512
207	0.512
208	0.512
209	0.513
210	0.513
211	0.515
212	0.516
213	0.518
214	0.519
215	0.519
216	0.520
217	0.521
218	0.522
219	0.523
220	0.525
221	0.525
222	0.525
223	0.526
224	0.526
225	0.530
226	0.533
227	0.536
228	0.537
229	0.537
230	0.537
231	0.538
232	0.541
233	0.543
234	0.543
235	0.544
236	0.544
237	0.545
238	0.545
239	0.546
240	0.547
241	0.547
242	0.548
243	0.548
244	0.549
245	0.549
246	0.551
247	0.551
248	0.552
249	0.552
250	0.552
251	0.553
252	0.553
253	0.553
254	0.554

Terr.	Rel. Factor
409	0.668
410	0.670
411	0.671
412	0.672
413	0.672
414	0.672
415	0.672
416	0.673
417	0.673
418	0.674
419	0.675
420	0.675
421	0.677
422	0.677
423	0.678
424	0.683
425	0.683
426	0.686
427	0.686
428	0.686
429	0.686
430	0.687
431	0.687
432	0.687
433	0.688
434	0.688
435	0.688
436	0.689
437	0.689
438	0.690
439	0.691
440	0.691
441	0.691
442	0.693
443	0.694
444	0.694
445	0.694
446	0.695
447	0.695
448	0.698
449	0.698
450	0.701
451	0.702
452	0.703
453	0.704
454	0.704
455	0.704
456	0.705
457	0.705
458	0.706

Terr.	Rel. Factor
613	0.835
614	0.835
615	0.835
616	0.836
617	0.838
618	0.838
619	0.842
620	0.842
621	0.843
622	0.846
623	0.846
624	0.850
625	0.851
626	0.855
627	0.856
628	0.858
629	0.859
630	0.860
631	0.862
632	0.863
633	0.864
634	0.865
635	0.865
636	0.869
637	0.872
638	0.874
639	0.881
640	0.882
641	0.883
642	0.884
643	0.886
644	0.887
645	0.887
646	0.889
647	0.890
648	0.890
649	0.891
650	0.891
651	0.892
652	0.893
653	0.893
654	0.894
655	0.895
656	0.896
657	0.898
658	0.900
659	0.903
660	0.905
661	0.905
662	0.908

Terr.	Rel. Factor
817	1.118
818	1.118
819	1.120
820	1.120
821	1.121
822	1.123
823	1.124
824	1.125
825	1.125
826	1.127
827	1.129
828	1.129
829	1.130
830	1.130
831	1.131
832	1.132
833	1.133
834	1.134
835	1.135
836	1.136
837	1.138
838	1.139
839	1.139
840	1.141
841	1.141
842	1.141
843	1.143
844	1.145
845	1.145
846	1.145
847	1.146
848	1.147
849	1.148
850	1.148
851	1.149
852	1.150
853	1.151
854	1.154
855	1.159
856	1.159
857	1.160
858	1.163
859	1.170
860	1.170
861	1.170
862	1.172
863	1.173
864	1.173
865	1.173
866	1.174

Terr.	Rel. Factor
1021	1.367
1022	1.371
1023	1.374
1024	1.375
1025	1.377
1026	1.381
1027	1.382
1028	1.382
1029	1.384
1030	1.388
1031	1.391
1032	1.392
1033	1.393
1034	1.394
1035	1.396
1036	1.398
1037	1.411
1038	1.413
1039	1.414
1040	1.414
1041	1.415
1042	1.418
1043	1.421
1044	1.421
1045	1.422
1046	1.423
1047	1.424
1048	1.425
1049	1.427
1050	1.427
1051	1.428
1052	1.430
1053	1.431
1054	1.432
1055	1.433
1056	1.435
1057	1.440
1058	1.442
1059	1.445
1060	1.446
1061	1.448
1062	1.451
1063	1.452
1064	1.453
1065	1.454
1066	1.455
1067	1.455
1068	1.457
1069	1.458
1070	1.458

Terr.	Rel. Factor
1225	1.773
1226	1.780
1227	1.780
1228	1.782
1229	1.782
1230	1.785
1231	1.786
1232	1.787
1233	1.787
1234	1.788
1235	1.789
1236	1.790
1237	1.791
1238	1.792
1239	1.793
1240	1.793
1241	1.795
1242	1.799
1243	1.801
1244	1.802
1245	1.804
1246	1.805
1247	1.805
1248	1.806
1249	1.808
1250	1.809
1251	1.815
1252	1.816
1253	1.821
1254	1.822
1255	1.825
1256	1.826
1257	1.829
1258	1.829
1259	1.830
1260	1.831
1261	1.839
1262	1.840
1263	1.845
1264	1.846
1265	1.850
1266	1.853
1267	1.854
1268	1.858
1269	1.859
1270	1.863
1271	1.866
1272	1.867
1273	1.869
1274	1.870

Terr.	Rel. Factor
51	0.319
52	0.320
53	0.320
54	0.321
55	0.322
56	0.322
57	0.324
58	0.327
59	0.328
60	0.328
61	0.329
62	0.329
63	0.332
64	0.335
65	0.335
66	0.336
67	0.337
68	0.337
69	0.338
70	0.338
71	0.341
72	0.342
73	0.343
74	0.357
75	0.347
76	0.350
77	0.354
78	0.355
79	0.355
80	0.356
81	0.357
82	0.359
83	0.360
84	0.361
85	0.361
86	0.362
87	0.362
88	0.362
89	0.363
90	0.366
91	0.366
92	0.371
93	0.373
94	0.374
95	0.376
96	0.379
97	0.380
98	0.381
99	0.381
100	0.382
101	0.387
102	0.387

Terr.	Rel. Factor
255	0.554
256	0.555
257	0.555
258	0.555
259	0.555
260	0.556
261	0.556
262	0.558
263	0.559
264	0.560
265	0.561
266	0.561
267	0.562
268	0.563
269	0.564
270	0.564
271	0.566
272	0.567
273	0.567
274	0.567
275	0.568
276	0.569
277	0.572
278	0.572
279	0.573
280	0.574
281	0.574
282	0.574
283	0.575
284	0.576
285	0.577
286	0.577
287	0.578
288	0.578
289	0.578
290	0.578
291	0.579
292	0.579
293	0.581
294	0.581
295	0.582
296	0.583
297	0.583
298	0.584
299	0.586
300	0.586
301	0.587
302	0.587
303	0.588
304	0.589
305	0.589
306	0.590

Terr.	Rel. Factor
459	0.706
460	0.708
461	0.709
462	0.709
463	0.710
464	0.711
465	0.711
466	0.711
467	0.711
468	0.712
469	0.715
470	0.716
471	0.717
472	0.717
473	0.718
474	0.718
475	0.718
476	0.719
477	0.720
478	0.720
479	0.722
480	0.723
481	0.726
482	0.727
483	0.727
484	0.727
485	0.728
486	0.728
487	0.728
488	0.728
489	0.731
490	0.734
491	0.734
492	0.735
493	0.735
494	0.736
495	0.738
496	0.739
497	0.739
498	0.740
499	0.741
500	0.742
501	0.742
502	0.743
503	0.745
504	0.745
505	0.747
506	0.747
507	0.749
508	0.749
509	0.749
510	0.749

Terr.	Rel. Factor
663	0.908
664	0.909
665	0.909
666	0.910
667	0.911
668	0.912
669	0.916
670	0.919
671	0.919
672	0.921
673	0.922
674	0.924
675	0.925
676	0.925
677	0.926
678	0.926
679	0.927
680	0.927
681	0.928
682	0.928
683	0.930
684	0.930
685	0.930
686	0.932
687	0.934
688	0.935
689	0.936
690	0.936
691	0.938
692	0.938
693	0.939
694	0.943
695	0.943
696	0.944
697	0.945
698	0.946
699	0.946
700	0.947
701	0.950
702	0.951
703	0.951
704	0.952
705	0.952
706	0.953
707	0.954
708	0.954
709	0.955
710	0.957
711	0.958
712	0.963
713	0.967
714	0.967

Terr.	Rel. Factor
867	1.175
868	1.176
869	1.177
870	1.181
871	1.181
872	1.182
873	1.182
874	1.183
875	1.183
876	1.184
877	1.186
878	1.187
879	1.188
880	1.195
881	1.195
882	1.196
883	1.196
884	1.197
885	1.198
886	1.198
887	1.199
888	1.202
889	1.202
890	1.204
891	1.205
892	1.206
893	1.208
894	1.209
895	1.211
896	1.212
897	1.212
898	1.212
899	1.215
900	1.215
901	1.216
902	1.220
903	1.221
904	1.221
905	1.222
906	1.222
907	1.223
908	1.223
909	1.223
910	1.223
911	1.225
912	1.225
913	1.227
914	1.228
915	1.230
916	1.231
917	1.232
918	1.233

Terr.	Rel. Factor
1071	1.461
1072	1.461
1073	1.461
1074	1.461
1075	1.463
1076	1.465
1077	1.470
1078	1.471
1079	1.471
1080	1.474
1081	1.478
1082	1.479
1083	1.480
1084	1.483
1085	1.484
1086	1.485
1087	1.490
1088	1.490
1089	1.491
1090	1.491
1091	1.492
1092	1.493
1093	1.494
1094	1.496
1095	1.497
1096	1.499
1097	1.502
1098	1.505
1099	1.507
1100	1.508
1101	1.509
1102	1.510
1103	1.515
1104	1.515
1105	1.518
1106	1.519
1107	1.520
1108	1.521
1109	1.522
1110	1.523
1111	1.523
1112	1.524
1113	1.525
1114	1.526
1115	1.529
1116	1.532
1117	1.534
1118	1.536
1119	1.538
1120	1.539
1121	1.539
1122	1.540

Terr.	Rel. Factor
1275	1.870
1276	1.873
1277	1.881
1278	1.885
1279	1.890
1280	1.891
1281	1.894
1282	1.895
1283	1.896
1284	1.897
1285	1.901
1286	1.905
1287	1.906
1288	1.909
1289	1.912
1290	1.913
1291	1.914
1292	1.916
1293	1.916
1294	1.921
1295	1.928
1296	1.928
1297	1.930
1298	1.935
1299	1.938
1300	1.941
1301	1.941
1302	1.943
1303	1.946
1304	1.949
1305	1.950
1306	1.951
1307	1.953
1308	1.960
1309	1.962
1310	1.963
1311	1.966
1312	1.970
1313	1.972
1314	1.974
1315	1.974
1316	1.988
1317	1.989
1318	1.991
1319	1.992
1320	1.999
1321	1.999
1322	2.000
1323	2.002
1324	2.002
1325	2.004
1326	2.004

Terr.	Rel. Factor	Terr.	Rel. Factor	Terr.	Rel. Factor	Terr.	Rel. Factor	Terr.	Rel. Factor	Terr.	Rel. Factor	Terr.	Rel. Factor
103	0.390	307	0.592	511	0.752	715	0.968	919	1.233	1123	1.541	1327	2.006
104	0.358	308	0.592	512	0.753	716	0.968	920	1.234	1124	1.542	1328	2.006
105	0.720	309	0.592	513	0.754	717	0.970	921	1.234	1125	1.543	1329	2.010
106	0.390	310	0.594	514	0.755	718	0.970	922	1.236	1126	1.548	1330	2.014
107	0.392	311	0.596	515	0.757	719	0.970	923	1.237	1127	1.548	1331	2.027
108	0.393	312	0.598	516	0.758	720	0.970	924	1.237	1128	1.553	1332	2.028
109	0.394	313	0.598	517	0.759	721	0.971	925	1.237	1129	1.554	1333	2.028
110	0.394	314	0.599	518	0.761	722	0.971	926	1.239	1130	1.555	1334	2.033
111	0.394	315	0.599	519	0.761	723	0.978	927	1.240	1131	1.556	1335	2.035
112	0.398	316	0.605	520	0.762	724	0.980	928	1.240	1132	1.558	1336	2.044
113	0.398	317	0.605	521	0.762	725	0.981	929	1.240	1133	1.562	1337	2.046
114	0.401	318	0.605	522	0.762	726	0.981	930	1.245	1134	1.563	1338	2.050
115	0.402	319	0.605	523	0.764	727	0.981	931	1.246	1135	1.564	1339	2.052
116	0.402	320	0.606	524	0.765	728	0.985	932	1.246	1136	1.567	1340	2.053
117	0.403	321	0.607	525	0.765	729	0.986	933	1.246	1137	1.569	1341	2.062
118	0.404	322	0.607	526	0.766	730	0.986	934	1.248	1138	1.575	1342	2.079
119	0.407	323	0.610	527	0.767	731	0.987	935	1.249	1139	1.579	1343	2.085
120	0.396	324	0.613	528	0.768	732	0.989	936	1.251	1140	1.580	1344	2.091
121	0.528	325	0.613	529	0.768	733	0.990	937	1.251	1141	1.584	1345	2.092
122	0.411	326	0.614	530	0.769	734	0.991	938	1.253	1142	1.590	1346	2.092
123	0.412	327	0.615	531	0.769	735	0.992	939	1.253	1143	1.591	1347	2.097
124	0.413	328	0.615	532	0.771	736	0.993	940	1.253	1144	1.592	1348	2.098
125	0.414	329	0.616	533	0.771	737	0.993	941	1.255	1145	1.596	1349	2.106
126	0.414	330	0.616	534	0.771	738	0.995	942	1.256	1146	1.597	1350	2.106
127	0.416	331	0.618	535	0.772	739	0.997	943	1.259	1147	1.597	1351	2.126
128	0.421	332	0.619	536	0.773	740	1.000	944	1.259	1148	1.603	1352	2.129
129	0.422	333	0.619	537	0.774	741	1.003	945	1.260	1149	1.603	1353	2.144
130	0.423	334	0.620	538	0.775	742	1.003	946	1.261	1150	1.610	1354	2.150
131	0.423	335	0.621	539	0.776	743	1.005	947	1.262	1151	1.614	1355	2.152
132	0.423	336	0.622	540	0.776	744	1.007	948	1.262	1152	1.616	1356	2.163
133	0.423	337	0.624	541	0.778	745	1.008	949	1.263	1153	1.617	1357	2.164
134	0.424	338	0.625	542	0.778	746	1.008	950	1.265	1154	1.625	1358	2.167
135	0.426	339	0.625	543	0.779	747	1.010	951	1.266	1155	1.625	1359	2.175
136	0.428	340	0.625	544	0.779	748	1.010	952	1.269	1156	1.626	1360	2.175
137	0.428	341	0.626	545	0.780	749	1.013	953	1.269	1157	1.630	1361	2.181
138	0.429	342	0.628	546	0.780	750	1.021	954	1.269	1158	1.630	1362	2.190
139	0.429	343	0.628	547	0.780	751	1.023	955	1.270	1159	1.630	1363	2.193
140	0.431	344	0.628	548	0.780	752	1.025	956	1.273	1160	1.630	1364	2.194
141	0.434	345	0.629	549	0.782	753	1.027	957	1.273	1161	1.632	1365	2.206
142	0.437	346	0.631	550	0.782	754	1.027	958	1.275	1162	1.633	1366	2.208
143	0.437	347	0.631	551	0.783	755	1.028	959	1.275	1163	1.633	1367	2.222
144	0.440	348	0.631	552	0.784	756	1.028	960	1.276	1164	1.635	1368	2.226
145	0.441	349	0.632	553	0.784	757	1.028	961	1.278	1165	1.645	1369	2.243
146	0.442	350	0.632	554	0.785	758	1.029	962	1.278	1166	1.648	1370	2.248
147	0.442	351	0.633	555	0.787	759	1.031	963	1.279	1167	1.649	1371	2.292
148	0.443	352	0.634	556	0.787	760	1.033	964	1.279	1168	1.649	1372	2.309
149	0.443	353	0.636	557	0.788	761	1.034	965	1.280	1169	1.651	1373	2.325
150	0.444	354	0.636	558	0.788	762	1.035	966	1.283	1170	1.653	1374	2.332
151	0.444	355	0.637	559	0.788	763	1.038	967	1.283	1171	1.654	1375	2.336
152	0.444	356	0.637	560	0.790	764	1.040	968	1.285	1172	1.654	1376	2.363
153	0.444	357	0.637	561	0.792	765	1.043	969	1.287	1173	1.661	1377	2.379
154	0.444	358	0.637	562	0.793	766	1.043	970	1.288	1174	1.662	1378	2.384

Terr.	Rel. Factor
155	0.444
156	0.445
157	0.445
158	0.446
159	0.446
160	0.447
161	0.448
162	0.450
163	0.452
164	0.454
165	0.457
166	0.457
167	0.458
168	0.458
169	0.458
170	0.460
171	0.463
172	0.465
173	0.466
174	0.468
175	0.468
176	0.468
177	0.474
178	0.474
179	0.475
180	0.476
181	0.477
182	0.477
183	0.477
184	0.479
185	0.480
186	0.481
187	0.482
188	0.483
189	0.485
190	0.486
191	0.492
192	0.493
193	0.497
194	0.497
195	0.498
196	0.498
197	0.498
198	0.501
199	0.502
200	0.506
201	0.506
202	0.507
203	0.510
204	0.511

Terr.	Rel. Factor
359	0.638
360	0.638
361	0.639
362	0.639
363	0.644
364	0.644
365	0.644
366	0.644
367	0.644
368	0.645
369	0.646
370	0.646
371	0.646
372	0.647
373	0.648
374	0.648
375	0.649
376	0.649
377	0.650
378	0.650
379	0.651
380	0.651
381	0.651
382	0.651
383	0.652
384	0.653
385	0.653
386	0.654
387	0.654
388	0.655
389	0.655
390	0.656
391	0.656
392	0.657
393	0.657
394	0.658
395	0.658
396	0.660
397	0.660
398	0.660
399	0.661
400	0.661
401	0.662
402	0.663
403	0.663
404	0.663
405	0.665
406	0.666
407	0.667
408	0.667

Terr.	Rel. Factor
563	0.793
564	0.793
565	0.794
566	0.794
567	0.795
568	0.795
569	0.796
570	0.796
571	0.796
572	0.797
573	0.798
574	0.798
575	0.798
576	0.802
577	0.802
578	0.804
579	0.804
580	0.804
581	0.805
582	0.805
583	0.806
584	0.806
585	0.806
586	0.807
587	0.808
588	0.808
589	0.808
590	0.809
591	0.810
592	0.810
593	0.810
594	0.812
595	0.813
596	0.814
597	0.816
598	0.818
599	0.818
600	0.819
601	0.819
602	0.819
603	0.823
604	0.823
605	0.824
606	0.826
607	0.826
608	0.827
609	0.829
610	0.829
611	0.831
612	0.834

Terr.	Rel. Factor
767	1.047
768	1.048
769	1.058
770	1.060
771	1.060
772	1.061
773	1.062
774	1.063
775	1.065
776	1.066
777	1.069
778	1.069
779	1.070
780	1.070
781	1.071
782	1.072
783	1.072
784	1.073
785	1.074
786	1.074
787	1.075
788	1.076
789	1.079
790	1.081
791	1.081
792	1.083
793	1.084
794	1.087
795	1.090
796	1.092
797	1.093
798	1.094
799	1.097
800	1.102
801	1.103
802	1.103
803	1.104
804	1.104
805	1.104
806	1.105
807	1.106
808	1.106
809	1.106
810	1.106
811	1.111
812	1.113
813	1.115
814	1.115
815	1.116
816	1.116

Terr.	Rel. Factor
971	1.288
972	1.290
973	1.291
974	1.291
975	1.291
976	1.292
977	1.292
978	1.292
979	1.292
980	1.293
981	1.294
982	1.294
983	1.296
984	1.296
985	1.298
986	1.298
987	1.299
988	1.301
989	1.305
990	1.308
991	1.311
992	1.314
993	1.315
994	1.316
995	1.317
996	1.320
997	1.322
998	1.323
999	1.324
1000	1.328
1001	1.331
1002	1.333
1003	1.334
1004	1.337
1005	1.338
1006	1.338
1007	1.343
1008	1.344
1009	1.346
1010	1.346
1011	1.349
1012	1.356
1013	1.358
1014	1.359
1015	1.360
1016	1.360
1017	1.361
1018	1.363
1019	1.364
1020	1.364

Terr.	Rel. Factor
1175	1.662
1176	1.666
1177	1.668
1178	1.675
1179	1.677
1180	1.678
1181	1.679
1182	1.689
1183	1.693
1184	1.695
1185	1.696
1186	1.697
1187	1.699
1188	1.699
1189	1.700
1190	1.700
1191	1.701
1192	1.702
1193	1.703
1194	1.703
1195	1.710
1196	1.710
1197	1.715
1198	1.719
1199	1.720
1200	1.721
1201	1.722
1202	1.723
1203	1.724
1204	1.727
1205	1.728
1206	1.729
1207	1.732
1208	1.733
1209	1.735
1210	1.737
1211	1.742
1212	1.744
1213	1.744
1214	1.744
1215	1.744
1216	1.748
1217	1.749
1218	1.755
1219	1.756
1220	1.757
1221	1.758
1222	1.762
1223	1.763
1224	1.766

Terr.	Rel. Factor
1379	2.391
1380	2.396
1381	2.401
1382	2.403
1383	2.425
1384	2.438
1385	2.438
1386	2.472
1387	2.496
1388	2.503
1389	2.510
1390	2.516
1391	2.521
1392	2.561
1393	2.571
1394	2.579
1395	2.624
1396	2.629
1397	2.636
1398	2.676
1399	2.676
1400	2.682
1401	2.713
1402	2.726
1403	2.737
1404	2.743
1405	2.753
1406	2.762
1407	2.763
1408	2.768
1409	2.788
1410	2.791
1411	2.792
1412	2.798
1413	2.801
1414	2.872
1415	2.951
1416	2.960
1417	3.069
1418	3.138
1419	3.162
1420	3.296
1421	3.433
1422	3.634
1423	3.874
1424	3.954
1425	4.750